Solutions to Selected Homework Problems
Math 9 — Fall 2005

§4.5 Summary of Curve Sketching

§4.5, #3.

$$\begin{aligned} f(x) &= 2 - 15x + 9x^2 - x^3 &&= -(x-2)(x^2 - 7x + 1) \\ f'(x) &= -15 + 18x - 3x^2 &&= -3(x-1)(x-5) \\ f''(x) &= 18 - 6x &&= 6(3-x) \end{aligned}$$

[A *Domain*] All x.
[B *Intercepts*] $f(0) = 2$. The x-intercepts are $x = 2$ and the roots of $x^2 - 7x + 1$, which are $x \approx 0.146$ and $x \approx 6.854$.
[C *Symmetry*] There is no obvious symmetry.
[D *Asymptotes*] There are no asymptotes.
[E *Intervals of Increase/Decrease*] Increasing for $1 < x < 5$, since that's where $f'(x) > 0$. Decreasing for $x < 1$ and for $x > 5$.
[F *Local Maxs and Mins*] Local min at $x = 1$ and local max at $x = 5$ by first derivative test.
[G *Concavity and Inflection Points*] Concave up for $x < 3$, since that's where $f''(x) > 0$. Concave down for $x > 3$. Inflection point at $x = 3$.

§4.5, #11.

$$f(x) = \frac{1}{x^2 - 9} \qquad f'(x) = \frac{-2x}{(x^2-9)^2} \qquad f''(x) = \frac{6(x^2+3)}{(x^2-9)^3}$$

[A *Domain*] All $x \neq \pm 3$.
[B *Intercepts*] $f(0) = -1/9$. There are no x-intercepts.
[C *Symmetry*] Symmetric about y-axis, since $f(x) = f(-x)$.
[D *Asymptotes*] Vertical asymptotes at $x = 3$ and $x = -3$. Horizontal asymptotes along the y-axis as $x \to \infty$ and as $x \to -\infty$. More precisely,

$$\begin{aligned} \lim_{x \to 3^+} f(x) &= \infty & \lim_{x \to 3^-} f(x) &= -\infty \\ \lim_{x \to -3^+} f(x) &= -\infty & \lim_{x \to -3^-} f(x) &= \infty \\ \lim_{x \to \infty} f(x) &= 0 & \lim_{x \to -\infty} f(x) &= 0 \end{aligned}$$

[E *Intervals of Increase/Decrease*] Increasing for all $x < 0$ and decreasing for all $x > 0$.
[F *Local Maxs and Mins*] Only critical number is $x = 0$. Since $f''(0) = -2/81 < 0$, the second derivative test tells us that $(0, -1/9)$ is a local maximum.
[G *Concavity and Inflection Points*] Concave up for $x < -3$ and for $x > 3$, concave down for $-3 < x < 3$. There are no inflection points.

§4.5, #12.

$$f(x) = \frac{x}{x^2 - 9} \qquad f'(x) = \frac{-(x^2+9)}{(x^2-9)^2} \qquad f''(x) = \frac{2x(x^2+27)}{(x^2-9)^3}$$

[A *Domain*] All $x \neq \pm 3$.
[B *Intercepts*] The only x or y intercept is the point $(0, 0)$.
[C *Symmetry*] The graph is symmetric about the origin, since $f(-x) = -f(x)$.
[D *Asymptotes*] Vertical asymptotes at $x = 3$ and $x = -3$. Horizontal asymptotes along the y-axis as $x \to \infty$ and as $x \to -\infty$. More precisely,

$$\lim_{x\to 3^+} f(x) = \infty \qquad \lim_{x\to 3^-} f(x) = -\infty$$
$$\lim_{x\to -3^+} f(x) = \infty \qquad \lim_{x\to -3^-} f(x) = -\infty$$
$$\lim_{x\to \infty} f(x) = 0 \qquad \lim_{x\to -\infty} f(x) = 0$$

[E *Intervals of Increase/Decrease*] The graph is decreasing for all values of x (other then $x = \pm 3$).
[F *Local Maxs and Mins*] There are no local maxima or minima.
[G *Concavity and Inflection Points*] Concave up for $-3 < x < 0$ and $x > 3$. Concave down for $x < -3$ and $0 < x < 3$. There is an inflection point at $x = 0$.

§4.5, #13.

$$f(x) = \frac{x}{x^2 + 9} \qquad f'(x) = \frac{9 - x^2}{(x^2+9)^2} \qquad f''(x) = \frac{2x(x^2-27)}{(x^2+9)^3}$$

[A *Domain*] All values of x.
[B *Intercepts*] The only x or y intercept is the point $(0, 0)$.

[C *Symmetry*] The graph is symmetric about the origin, since $f(-x) = -f(x)$.
[D *Asymptotes*] The x-axis is a horizontal asymptote, since $\lim_{x\to\pm\infty} f(x) = 0$.
[E *Intervals of Increase/Decrease*] Increasing for $-3 < x < 3$. Decreasing for $x < -3$ and for $x > 3$.
[F *Local Maxs and Mins*] Critical values are $x = \pm 3$. By first derivative test, $x = -3$ gives a local min and $x = 3$ gives a local max. Or can use second derivative test, $f''(-3) = 1/54 > 0$ and $f''(3) = -1/54 < 0$.
[G *Concavity and Inflection Points*] The concavity is given by

$x < -3\sqrt{3}$	Concave Down
$-3\sqrt{3} < x < 0$	Concave Up
$0 < x < 3\sqrt{3}$	Concave Down
$3\sqrt{3} < x$	Concave Up

There is an inflection point at $x = 0$.

§4.5, #14.

$$f(x) = \frac{x^2}{x^2+9} \qquad f'(x) = \frac{18x}{(x^2+9)^2} \qquad f''(x) = \frac{54(3-x^2)}{(x^2+9)^3}$$

One way to simplify this problem is to notice that

$$f(x) = \frac{x^2}{x^2+9} = \frac{x^2+9-9}{x^2+9} = 1 - \frac{9}{x^2+9}.$$

So you can first graph $\frac{9}{x^2+9}$, then reflect it about the x-axis and move it up one unit. But we'll do the problem without using this trick.
[A *Domain*] All x.
[B *Intercepts*] The only x or y intercept is the point $(0,0)$.
[C *Symmetry*] Symmetric about the y-axis, since $f(-x) = f(x)$.
[D *Asymptotes*] There are no vertical asymptotes. The line $y = 1$ is a horizontal asymptote, since $\lim_{x\to\pm\infty} f(x) = 1$.
[E *Intervals of Increase/Decrease*] Increasing for $x > 0$. Decreasing for $x < 0$.
[F *Local Maxs and Mins*] Critical point at $x = 0$. Since $f''(0) > 0$, the point $(0,0)$ is a local minimum.

[G *Concavity and Inflection Points*] Concave up for $-\sqrt{3} < x < \sqrt{3}$. Concave down for $x < -\sqrt{3}$ and $x > \sqrt{3}$. Inflection points at $x = -\sqrt{3}$ and $x = \sqrt{3}$.

§4.5, #49.

$$f(x) = xe^{-x^2} \qquad f'(x) = (1 - 2x^2)e^{-x^2} \qquad f''(x) = 2x(2x - 3)e^{-x^2}$$

[A *Domain*] All x.
[B *Intercepts*] The only x or y intercept is the point $(0, 0)$.
[C *Symmetry*] The graph is symmetric about the origin, since $f(-x) = -f(x)$.
[D *Asymptotes*] There are no vertical asymptotes. The y-axis is a horizontal asymptote, since $\lim_{x\to\pm\infty} f(x) = 0$.
[E *Intervals of Increase/Decrease*] Increasing for $-1/\sqrt{2} < x < 1/\sqrt{2}$. Decreasing for $x < -1/\sqrt{2}$ and for $x > 1/\sqrt{2}$.
[F *Local Maxs and Mins*] Local min at $x = -1/\sqrt{2}$ and local max at $x = 1/\sqrt{2}$, by first derivative test.
[G *Concavity and Inflection Points*] Concave up for $x < 0$ and $x > 3/2$. Concave down for $0 < x < 3/2$. Inflection points for $x = 0$ and $x = 3/2$.