Math 1560 Research Project
Due Friday, March 19 at 10:00am
Late assignments will not be accepted.
READ THE INSTRUCTIONS CAREFULLY

Instructions:

- The goal of this project is for you to do original mathematical research. A paradigm for doing research includes the following steps:
  - Gather data.
  - Look for patterns, make conjectures.
  - Test your conjectures against further data.
  - Prove some of your conjectures.
  - If unable to prove a conjecture, try to do a special case, or an analogous problem. (Sometimes it's helpful to generalize a conjecture, because it may be easier to prove a stronger statement!)
- You are to do this project **entirely on your own**. You may not consult with any other people. You may not look for information on the web.
- You may use a programmable calculator or computer to generate data as long as you write the program yourself. (Do not include the program in your report, but do mention that you wrote and used it.)
- There are no right or wrong answers. (However, there are true statements and false statements; try to avoid making any of the latter sort. Of course, it's fine to make conjectures that might be false.)
- Your project should either be typed (12 point type) or neatly handwritten. Spelling, grammar, and clarity count, so be sure to proofread your work before submitting it.
- Try to include conjectures, theorems (with clearly written proofs), and data and examples illustrating your discoveries.
- **Maximium length is five pages**. Shorter is acceptable. You'll get a much better grade for two pages with actual content than you will for five pages of garbage.

Description of the MA 1560 Project

Let $a \geq 1$ and $b \geq 1$ be positive integers satisfying

$$\gcd(a, b) = 1.$$

Then we know that every integer n can be expressed in the form

$$ax + by = n \quad \text{for some } x, y \in \mathbb{Z}.$$

However, if we require x and y to be non-negative, then there are often positive integers that cannot be expressed in this way.

For example, take $a = 4$ and $b = 5$. Then the expression

$$4x + 5y \quad \text{with } x, y \geq 0$$

is never equal to 3 or 7 or 17, among other values.

Your task is to investigate which positive integers cannot be expressed in the form $ax + by$ with $x, y \geq 0$, i.e., the numbers in the set

$$\mathcal{T}(a, b) = \{n \in \mathbb{N} : n \neq ax + by \text{ for all } x, y \in \mathbb{Z} \text{ with } x, y \geq 0\}.$$

To get you started, here are a few questions that you might try to answer. (But I expect you to come up with other questions of your own.)

1. Is the set $\mathcal{T}(a, b)$ finite or infinite?
2. If $\mathcal{T}(a, b)$ is finite, what does its largest element look like? This could mean finding an upper bound, or maybe even finding an exact formula.
3. If $\mathcal{T}(a, b)$ is finite, how many elements does it contain? Again, you could find an upper bound, or maybe an exact formula.
4. Generalize, for example to $ax + by + cz$ for positive integers a, b, c satisfying $\gcd(a, b, c) = 1$.