Selectians

Group Work 7

1. Compute the derivatives of the following functions.

   (a) \( \sin(x) \)
   \[ \cos x \]

   (b) \( \sin(x^5) \)
   \[ \cos(x^5) \cdot 5x^4 \]

   (c) \( \cos(x/2) \)
   \[ -\sin(x/2) \cdot \frac{1}{2} \]

   (d) \( \cos(x) \cdot \sin(x + 2) \)
   \[ 
   \cos x \cdot \frac{d}{dx} \sin(x+2) + \left( \frac{d}{dx} (\cos x) \right) \cdot \sin(x+2) 
   \]
   \[ 
   = \cos x \cdot \cos(x+2) - \sin x \cdot \sin(x+2) 
   \]

   (e) \( \cos(\sin(x)) \)
   \[ -\sin(\sin x) \cdot \cos x \]
2. Evaluate the following limits.

(a) \( \lim_{x \to \pi} \pi + \cos(\pi/2) = \pi + 0 = \pi \)

(b) \( \lim_{x \to 0} \frac{\sin(x^2)}{x^2} \)

let \( y = x^2 \)

\( \lim_{x \to 0} \frac{\sin(x^2)}{x^2} = \lim_{y \to 0} \frac{\sin(y)}{y} = 1. \)

(c) \( \lim_{x \to 0} \frac{\sin(3x)}{x} \)

\( = \lim_{x \to 0} 3 \cdot \frac{\sin(3x)}{3x} = 3 \cdot \lim_{x \to 0} \frac{\sin(3x)}{3x} = 3 \cdot 1 = 3 \)

(d) \( \lim_{x \to 0} \frac{\sin(x^3)}{x} \)

\( = \lim_{x \to 0} x^2 \cdot \frac{\sin(x^3)}{x^3} = \lim_{x \to 0} x^2 \cdot \lim_{x \to 0} \frac{\sin(x^3)}{x^3} = 0 \cdot 1 = 0 \)

(e) \( \lim_{x \to 0} \frac{\cos(x) - 1}{x} \)

\( \lim_{x \to 0} \frac{\cos(x) - 1}{x} = 1-1=0, \quad \lim_{x \to 0} x = 0, \quad \) so \( 2' \text{ or } \frac{15}{15} \)

\( \frac{\text{\textbf{\textit{rule}}}}{= \frac{1}{2}} \) \( \lim_{x \to 0} \frac{\cos(x) - 1}{x} = \lim_{x \to 0} \frac{-\sin(x)}{1} = \frac{-\sin(0)}{1} = 0 \)
Solutions

Name:

Math 60, Quiz 7.

Show all your work for full credit.

1. Evaluate the following limits or explain why the limit does not exist.

   (a) \( \lim_{x \to 0} \sin(1/x) \) \text{ does not exist.}  

      As \( x \to 0 \), \( 1/x \to \infty \) and \( \sin(1/x) \) takes on all values between \(-1\) and \(1\), repeatedly.

      \( \Rightarrow \) See also handout on \( \lim_{x \to 0} \sin(1/x) \).

   (b) \( \lim_{x \to 0} \frac{\sin(x)}{\sqrt{x}} \) (hint: this is 11.2, \#32 from this week's homework)

      \[
      \lim_{x \to 0} \frac{\sin(x)}{\sqrt{x}} = \lim_{x \to 0} \frac{x^{2/3} \sin x}{x} \\
      = \lim_{x \to 0} x^{2/3} \cdot \lim_{x \to 0} \frac{\sin x}{x} \\
      = 0 \cdot 1 = 0.
      \]
2. Compute the following derivatives.

(a) \( \frac{d}{dx}(\cos(x^3)) = -\sin(x^3) \cdot 3x^2 \)

(b) \( \frac{d}{dx}(\cos^3(x)) = 3\cos^2(x) \cdot -\sin x \)

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