Group Work 8

1. Complete the square.
   (a) \( x^2 + 4x + 5 \)
   \[
   \frac{b}{2} = 2 \\
   (x + 2)^2 = x^2 + 4x + 4 \quad \text{so} \quad x^2 + 4x + 5 = (x + 2)^2 + 1
   \]

   (b) \( x^2 - 2x - 3 \)
   \[
   \frac{b}{2} = -1 \\
   (x - 1)^2 - 4
   \]

   (c) \(-x^2 + 5x\)
   \[
   = -\left(x^2 - 5x\right) = -\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 \\
   = \left(\frac{5}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2
   \]

2. For \( x \) between \(-1/2\) and \(1/2\), is \( \sin^{-1}(x) \) increasing or decreasing? Why?

   As \( x \) goes from \(-1/2\) to \(1/2\),
   \( \sin^{-1}(x) \) goes from \(-\pi/6\) to \(\pi/6\), increasing.
   Also \( \frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}} \), which is positive for \( x \) between \(-1/2\) and \(1/2\).

3. Compute the following integrals.
   (a) \( \int_{0}^{\pi/2} \sin(2x) \, dx \)
   
   Let \( u = 2x \), \( du = 2 \, dx \)
   
   \[
   \int_{0}^{\pi/2} \sin(2x) \, dx = \int \sin u \cdot \frac{1}{2} \, du = \frac{1}{2} \left[ \sin u \right]_{0}^{\pi} \\
   = \frac{1}{2} \left[ -\cos(\pi) - (-\cos(0)) \right] = \frac{1}{2} (1 + 1) = 1
   \]
(b) \[ \int \cos(x) \cdot e^{\sin(x)} \, dx \]
let \( u = \sin(x) \), \( du = \cos(x) \, dx \)
\[ = \int e^u \, du = e^u + C = e^{\sin(x)} + C \]

(c) \[ \int \frac{1}{\sqrt{6x-x^2}} \, dx \]
\[ = \int \frac{1}{\sqrt{9-(x-3)^2}} \, dx \]
let \( u = x-3 \), \( du = dx \)
\[ = -\left( \left( x-3 \right)^2 - 9 \right) \]
\[ = 9 - (x-3)^2 \]
\[ \int \frac{1}{\sqrt{9^2-u^2}} \, du = \sin^{-1}\left( \frac{u}{3} \right) + C = \sin^{-1}\left( \frac{x-3}{3} \right) + C \]

(d) \[ \int \frac{1}{x^2+4x+5} \, dx \]
\[ x^2 + 4x + 5 = (x+2)^2 + 1 \]
\[ \int \frac{1}{x^2+4x+5} \, dx = \int \frac{1}{(x+2)^2+1} \, dx \]
let \( u = x+2 \), \( du = dx \)
\[ = \int \frac{1}{u^2+1} \, du \]
\[ = \tan^{-1}(u) + C \]
\[ = \tan^{-1}(x+2) + C \]
Name: Solutions

Math 60, Quiz 8.

Show all your work for full credit.

1. Evaluate the following limit or explain why it does not exist.
   (a) \( \lim_{x \to 0} \sin(1/x) \)
   See quiz 7 solutions

2. Compute the following integrals.
   (a) \( \int_0^{2\pi} \cos(2x) \, dx \)
   Let \( u = 2x, \, du = 2 \, dx \)
   \[
   = \int_{u=0}^{u=4\pi} \cos u \cdot \frac{1}{2} \, du = \frac{1}{2} \sin u \bigg|_0^{4\pi}
   = \frac{1}{2} (\sin 4\pi - \sin 0)
   = \frac{1}{2} (0 - 0) = 0.
   
1
(b) \[ \int \sin(x) \cdot e^{\cos(x)} \, dx \]

Let \( u = \cos(x) \), \( du = -\sin(x) \, dx \)

\[ = \int e^u \cdot (-1) \, du = -e^u + C = -e^{\cos(x)} + C \]

(c) \[ \int \frac{1}{2x^2 + 8x + 10} \, dx \]

\[ = \frac{1}{2} \int \frac{1}{x^2 + 4x + 5} \, dx \]

\[ = \frac{1}{2} \int \frac{1}{(x+2)^2 + 1} \, dx \]

Let \( u = x+2 \)

\[ = \frac{1}{2} \int \frac{1}{u^2 + 1} \, du \]

\[ = \frac{1}{2} \tan^{-1}(u) + C \]

\[ = \frac{1}{2} \tan^{-1}(x+2) + C \]