Divide the interval \([0, 1]\) into 4 subintervals of equal length.

- The sum of the areas of the 4 rectangles are:

\[
\frac{1}{4} \left( \frac{1}{16} \right) + \frac{1}{4} \left( \frac{4}{16} \right) + \frac{1}{4} \left( \frac{9}{16} \right) + \frac{1}{4} (1)
\]

\[
= \frac{1}{4} \left( \frac{1}{16} + \frac{4}{16} + \frac{9}{16} + \frac{16}{16} \right)
\]

\[
= \frac{1}{4} \left( \frac{30}{16} \right) = \frac{30}{64} = \frac{15}{32}
\]

\[
\text{More than actual area}
\]

**HW**: 4th update: \( \int k(x) \, dx = 1c \text{ Strada} \)

Sum of areas of rectangles is:

\[
\frac{1}{4} \left( 0 + \frac{1}{16} + \frac{4}{16} + \frac{9}{16} \right)
\]

\[
= \frac{1}{4} \left( \frac{14}{16} \right) = \frac{14}{64} = \frac{7}{32}
\]

We have used right hand end points here for the heights of our rectangles.

Here we use left hand endpoints.
How do we get a better approximation of the area? Use more subintervals:

\[ y = x^2 \]

Divide \([a, b]\) into \(n\) subintervals of length \(\frac{b-a}{n}\)

Using right hand endpoints for the heights of our rectangles, we get:

\[ \frac{1}{n} \left( \frac{1}{n} \right)^2 + \frac{1}{n} \left( \frac{2}{n} \right)^2 + \frac{1}{n} \left( \frac{3}{n} \right)^2 + \cdots + \frac{1}{n} \left( \frac{n}{n} \right)^2 = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{i}{n} \right)^2 \]

The sum of the areas of these rectangles is

\[ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{i}{n} \right)^2 = \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6n^2} = \frac{n(n+1)(2n+1)}{6n^3} \]

We define the area under \( y = x^2 \) from \( x = 0 \) to \( x = 1 \) to be

\[ \lim_{n \to \infty} \frac{n(n+1)(2n+1)}{6n^3} = \frac{1}{3} \]

(we computed this earlier in class).

If we used left hand endpoints, we would get the same answer in the end with the limit...