Solving trigonometric equations

There are many techniques you can use, so be sure to try all of them!

Ex. Collect like terms, then solve:

\[ \tan x = 3 \sqrt{3} - 2 \tan x \]
\[ 3 \tan x = 3 \sqrt{3} \]
\[ \tan x = \sqrt{3} \]

The function \( y = \tan x \) has period \( \pi \):

for \( x \) in \([0, \pi] \), i.e. \( 0 \leq x < \pi \),
\( \tan x = \sqrt{3} \) when \( x = \frac{\pi}{3} \).

General solution: \[ x = \frac{\pi}{3} + \pi n \]

Ex. Factor, then solve:

\[ 2 \cos^2 x \sin^2 x = \sin^2 x \]
\[ 2 \cos^2 x \sin^2 x - \sin^2 x = 0 \]
\[ \sin^2 x (2 \cos^2 x - 1) = 0 \]
\[ \sin^2 x = 0 \]
\[ \sin x = 0 \]
\( x = 0, \pi \) for \( x \) in \([0, 2\pi] \)

General solution: \( x = 0 + 2\pi n, x = \pi + 2\pi n \)

In total: \( x = \pi n \)
\[
\cos x = \pm \frac{1}{\sqrt{2}}
\]

\[
\cos x = \frac{1}{\sqrt{2}}
\]

\[
x = \frac{\pi}{4}, \frac{7\pi}{4} \quad \text{in } [0, 2\pi)
\]

General solution: \(x = \frac{\pi}{4} + 2\pi n\),

\[
x = \frac{7\pi}{4} + 2\pi n
\]

Collecting these two:

\[
x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \ldots
\]

so \(x = \frac{\pi}{4} + \frac{\pi}{2} n\)

Total solution: \(x = \frac{\pi}{4} + \frac{\pi}{2} n \) and \(x = \pi n\)

Ex. Factoring an equation of quadratic type:

\[
\cos^2 x - 3\cos x + 2 = 0
\]

\[
(\cos x - 1)(\cos x - 2) = 0
\]

\[
\begin{align*}
\cos x &= 1 \\
\cos x &= 2
\end{align*}
\]

\[
x = 0 \quad \text{in } [0, 2\pi)
\]

\[
\text{No solution because the range of } y = \cos x \text{ is } [-1, 1]
\]

\[
\text{and 2 is not in } [-1, 1]
\]

General solution: \(x = 2\pi n\)

Ex. Rewrite with a single trig function, then solve:

\[
-\sin^2 x - 3\cos x + 3 = 0
\]

\[
-(1 - \cos^2 x) - 3\cos x + 3 = 0
\]

\[
-1 + \cos^2 x - 3\cos x + 3 = 0
\]

\[
\cos^2 x - 3\cos x + 2 = 0
\]

\[
\text{Solve as above.}
\]

To make it easier, you can substitute \(u = \cos x\). Then

\[
u^2 - 3u + 2 = 0
\]

\[
(u - 1)(u - 2) = 0
\]

Substitute both in

\[
(\cos x - 1)(\cos x - 2) = 0
\]

• Can rewrite \(\sin^2 x\) as \(1 - \cos^2 x\)

• But rewriting \(\cos x\) as \(\sqrt{1 - \sin^2 x}\) introduces square roots and gets messy
Trick for Memorizing the Unit Circle

As you go around the unit circle, the number in the square root either decreases or increases by 1.

You should also understand the symmetry of the unit circle, reflecting over the X- and Y-axes (and over the lines \( y=x \) (that corresponds to \( \pi/4 \) and \( 5\pi/4 \)) and \( y=-x \) (that corresponds to \( 3\pi/4 \) and \( 7\pi/4 \))).