Name: Solutions

Math 60, Midterm 2.

Show all your work for full credit. 52 points total.

1. Circle T for true, F for false. For this question only, no work is required. (2 points each)
   (a) T  Let $f(x) = \sin(3x)$. The range of $f$ is $[-3, 3]$.
   (b) F  The equation $\cos x = \frac{1}{2}$ has infinitely many solutions.
   (c) F  If $x$ is between $\frac{\pi}{4}$ and $\frac{\pi}{2}$, then $\sin x \geq \cos x$.
   (d) F  For any value of $x$ where $\tan(x)$ is defined, $\tan(x) = \tan(x + \pi)$.
   (e) T, $\lim_{x \to \infty} \sin x = 0$. The limit does not exist. Look at the graph of $y = \sin x$.

2. (a) Using the formula for $\cos(u - v)$, verify that $\cos(\pi - v) = -\cos v$. (4 points)
   \[
   \cos(u - v) = \cos u \cos v + \sin u \sin v
   \]
   \[
   \cos(\pi - v) = \cos \pi \cos v + \sin \pi \sin v
   \]
   \[
   = -1 \cdot \cos v + 0 \cdot \sin v
   \]
   \[
   = -\cos v
   \]
(b) If \( v \) is between 0 and \( \frac{\pi}{2} \), draw a picture with the unit circle that illustrates why the identity from part 2a makes sense. (2 points)

\[
\begin{align*}
\cos v &= x \\
\cos(\pi - v) &= -x \\
&= -\cos v
\end{align*}
\]

3. Compute the following expressions.

(a) \( \cos 0 \) (1 point)

\[
\cos 0 = 1
\]

(b) \( \cos^{-1}(-1/2) \) (3 points)

\[
\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)
\]

(c) \( \tan \cos^{-1}(12/13) \) (4 points)

\[
\cos \theta = \frac{12}{13},
\cos^{-1}(12/13) = \theta
\]

\[
\tan \theta = \frac{5}{12}
\]
(d) $\sin^2(\frac{\pi}{4})$ (4 points)

\[
\sin^2(\frac{\pi}{4}) = \frac{1 - \cos(\frac{\pi}{4})}{2}
\]

\[
= \frac{1 - \frac{\sqrt{2}}{2}}{2}
\]

4. Sketch the following graphs. Your graphs should show all $x$-values between $-2\pi$ and $2\pi$. (3 points each)

(a) $y = \sin x$

(b) $y = \sin(x - \pi) \Rightarrow$ shift part a) by $\pi$ to the right
5. Solve the following equation for \( x \). Your answer should describe all possible solutions. (6 points)

\[ \sec^2(x) - \sec(x) - 2 = 0 \]

\[ (\sec x - 2)(\sec x + 1) = 0 \]

\[ \Rightarrow \sec x = 2 \quad \text{or} \quad \sec x = -1 \]

\[ \cos x = \frac{1}{2} \]

\[ x = \frac{\pi}{3} + 2\pi n, \quad x = -\frac{\pi}{3} + 2\pi n \]
6. (a) Draw a picture of an isosceles triangle for which the two legs have length 5 and the angle between the two legs is 50 degrees (recall that an isosceles triangle has two sides of the same length, called the legs of the triangle). (1 point)

(b) Compute the area of your triangle in part 6a. Your answer can involve trig functions with angles. (5 points)

\[
\text{Area} = \frac{1}{2}bh \\
\sin 25^\circ = \frac{\frac{1}{2}b}{5} \quad \Rightarrow \quad \frac{1}{2}b = 5 \sin 25^\circ \\
\cos 25^\circ = \frac{h}{5} \quad \Rightarrow \quad h = 5 \cos 25^\circ \\
\Rightarrow \quad \frac{1}{2}bh = 25 \cos 25^\circ \sin 25^\circ.
\]