

Calculus Review Problems for Probability and Statistics

Directions : Calculus is an essential tool in probability and statistics. These questions are designed to ensure that you have a sufficient mastery of the subject for the course. You must correctly solve these problems to continue with the course.

1 Derivatives (one variable)

Question 1. Find the derivative of $f(x) = 4x^5 + 3x^2 + x^{1/3}$.

Question 2. Find the derivative of $f(x) = (x^4 + 3x^2 + 8) \cos x$.

Question 3. Find the derivative of $f(x) = \log(1 - x^2)$.

Question 4. Find the derivative of $\log(4x) - \log(2x)$.

Question 5. Find the derivative of $e^{-x^2/2} = \exp(-x^2/2)$.

Question 6. Find the second derivative of $e^{-x^2/2} = \exp(-x^2/2)$.

Question 7. Find the derivative of $e^{x^8} \cos(3x^4) = \exp(x^8) \cos(3x^4)$.

Question 8. Find the derivative of the function $f(x) = 4x + \sqrt{2} \cos(x)$ and then use it to find the tangent line to the curve $y = f(x)$ at $x = \pi/4$. Use the tangent line to approximate $f(x)$ when $x = \frac{\pi}{4} + .01$.

Question 9. Find the second derivative of $f(x) = \ln x + \sqrt{162}$.

Question 10. Find the maximum value of $x^4 e^{-x} = x^4 \exp(-x)$ when $x \geq 0$.

Question 11. Find the critical points of $f(x) = 4x^3 + 3x^2$, and decide whether each is a maximum, a minimum, or a point of inflection.

Question 12. Find the derivative of $(x^2 - 1)/(x - 1)$.

Question 13. Find the derivative of the function $f(x) = \sqrt[3]{(x - 2)^2} = (x - 2)^{2/3}$.

Question 14. Find the points on the graph of $f(x) = \frac{1}{3}x^3 + x^2 - x - 1$ where the slope is (a) -1 , (b) 2 , and (c) 0 .

Question 15. Find the second derivative of $f(x) = (x^4 + 3x^2 + 8) \cos x$.

2 Taylor Series (one variable)

Question 16. Find the first five terms of the Taylor Series for $f(x) = x^8 + x^4 + 3$ at $x = 0$.

Question 17. Find the first three terms of the Taylor Series for $f(x) = x^8 + x^4 + 3$ at $x = 1$.

Question 18. Find the first three terms of the Taylor Series for $f(x) = \cos(5x)$ at $x = 0$.

Question 19. Find the first five terms of the Taylor Series for $f(x) = \cos^3(5x)$ at $x = 0$.

Question 20. Find the first two terms of the Taylor Series for $f(x) = e^x$ at $x = 0$.

Question 21. Find the first six terms of the Taylor Series for $f(x) = e^{x^8} = \exp(x^8)$ at $x = 0$.

Question 22. Find the first four terms of the Taylor series for $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2} = \exp(-x^2/2)/\sqrt{2\pi}$ at $x = 0$.

Question 23. Find the first three terms of the Taylor series for $f(x) = \sqrt{x}$ at $x = \frac{1}{3}$.

Question 24. Find the first three terms of the Taylor series for $f(x) = (1+x)^{1/3}$ at $x = \frac{1}{2}$.

Question 25. Find the first three terms of the Taylor series for $f(x) = x \log x$ at $x = 1$.

Question 26. Find the first three terms of the Taylor series for $f(x) = \log(1+x)$ at $x = 0$.

Question 27. Find the first three terms of the Taylor series for $f(x) = \log(1-x)$ at $x = 1$.

Question 28. Find the first two terms of the Taylor series for $f(x) = \log((1-x) \cdot e^x) = \log((1-x) \cdot \exp(x))$ at $x = 0$.

Question 29. Find the first three terms of the Taylor Series for $f(x) = \cos(x) \log(1+x)$ at $x = 0$.

Question 30. Find the first two terms of the Taylor series for $f(x) = \log(1+2x)$ at $x = 0$.

3 Integrals (one variable)

Question 31. Find the following integral: $\int_0^1 (x^4 + x^2 + 1)dx$.

Question 32. Find the following integral: $\int_0^1 (x^2 + 2x + 1)dx$.

Question 33. Find the following integral: $\int_0^1 (x^2 + 2x + 1)^2 dx$.

Question 34. Find the following integral: $\int_{-\pi/2}^{\pi/2} (\sin^3 x \cos x + \sin x \cos x) dx$.

Question 35. Find the following integral: $\int_{-4}^4 (x^3 + 6x^2 - 2x - 3) dx$.

Question 36. Find the following integral: $\int_0^1 \frac{x}{1+x^2} dx$.

Question 37. Find the following integral: $\int_0^3 (x^3 + 3x)^8 (x^2 + x) dx$.

Question 38. Find the following integral: $\int_0^2 x \cos(3x^2) dx$.

Question 39. Find the following integral: $\int_0^\infty x e^{-x^2/4} dx$.

Question 40. Let

$$f(x) = \begin{cases} 1 & \text{if } x \in [0, 1] \\ 0 & \text{otherwise.} \end{cases}$$

Calculate $\int_{-\infty}^\infty f(t)f(x-t)dt$.

4 Derivatives (several variables)

Question 41. Let $f(x) = x^2y + e^x + \sin(xy)$. Find $\partial f/\partial x$ and $\partial f/\partial y$.

Question 42. Let

$$f(x; \mu, \sigma) = \frac{\exp(-(x - \mu)^2/2\sigma^2)}{\sqrt{2\pi\sigma^2}}. \quad (4.1)$$

Find $\partial f/\partial \mu$ and $\partial f/\partial \sigma$.

Question 43. Find $\partial f/\partial x$ and $\partial f/\partial y$ for the function $f(x, y) = x e^{x^2+y^2} = x \exp(x^2 + y^2)$.

Question 44. Find $\partial f/\partial x$ and $\partial f/\partial y$ for the function $f(x, y) = e^{xy} - \log(x^2 + y^2)$.

Question 45. Find $\partial f/\partial x$ and $\partial f/\partial y$ for the function $f(x, y, t) = 5t^4 - 4t^5 \cos(t \sin t)$.

5 Integrals (several variables)

Question 46. Find

$$\int_{x=0}^2 \int_{y=0}^3 5(x^2y + xy^2 + 2) dx dy.$$

Question 47. Find

$$\int_{x=0}^6 \int_{y=0}^5 x e^{-xy} dx dy.$$

Question 48. Find

$$\int_{x=0}^1 \int_{y=0}^1 x^m y^n dx dy,$$

where $m, n > 0$.

Question 49. Find

$$\int_{x=0}^1 \int_{y=0}^1 (x^2 + 2xy + y\sqrt{x}) dx dy.$$

Question 50. Find

$$\int_{x=0}^1 \int_{y=0}^1 (ax + by + c) dx dy.$$