1. Dummit and Foote pp. 277-278 problems 1(a), 1(b), 3

2. Let $K$ be a field. Prove that $K[[x]]$ is a Euclidean domain with respect to the following norm: $N(0) = 0$, and for all nonzero $p \in K[[x]]$, $N(p)$ is the order of $p$, i.e. the smallest exponent appearing in $p$.

3. Dummit and Foote p. 283 problem 5

4. Compute a gcd of $4 + 2i$ and $5i$ in $\mathbb{Z}[i]$. Identifying $\mathbb{Z}[i]$ with the integer lattice points in the complex plane, draw a picture of the elements of the ideal $(4 + 2i, 5i)$.

5. Let $R$ be an integral domain. We defined the field of fractions $K$, whose elements are equivalence classes of $\{(a, b): a, b \in R, b \neq 0\}$ where $(a, b) \sim (c, d)$ if $ad = bc$. We write $a/b$ for the class of $(a, b)$. We defined

   \[ a/b + c/d = (ad + bc)/bd, \quad a/b \cdot c/d = (ac)/(bd). \]

Convince yourself that $+$ and $\cdot$ are well-defined and make $K$ into a field with $0 = 0/1$ and $1 = 1/1$ (ungraded).

(a) Prove that the map $i: R \to K$ given by $i(r) = r/1$ is a ring homomorphism sending all nonzero elements to units.

(b) Prove the following universal property of localization: Let $S$ be a commutative ring with 1. If $f: R \to S$ is any ring homomorphism sending all nonzero elements of $R$ to units of $S$, then there is a unique ring homomorphism $\tilde{f}: K \to S$ such that

   \[ f = \tilde{f} \circ i. \]