MATH 1530 ABSTRACT ALGEBRA
PROBLEM SET 6, DUE TUESDAY MARCH 14 1PM IN CLASS
worth 1/2 a problem set

1. An element g of a group G is called *torsion* if it has finite order, and G is called *torsion-free* if its only torsion element is the identity.

   Let A be an abelian group and let N be the set of its torsion elements. Prove that N is a subgroup and that A/N is torsion-free.

2. (No proofs necessary) Let $N = Z(D_{12}) \trianglelefteq D_{12}$.

   (a) List the elements of N. You may use Dummit and Foote Problem 4 on p.28, on Homework 3, or combine your answers from Problems 5 and 6 from Homework 5.
   Now list the elements of D_{12}/N; there should be six of them.

   (b) Write out the multiplication table for the group D_{12}/N. This should be a 6×6 table, all of whose entries are taken from your list of elements of D_{12}/N from part (a).

3. Number the three diagonals of a regular hexagon as shown. Let D_{12} act on the set $\{1, 2, 3\}$ via $g \cdot i = j$ if $g \in D_{12}$ takes the diagonal i to diagonal j.

   Let $\phi\colon D_{12} \to S_3$ be the permutation representation of this action. Use ϕ to prove that

$$D_{12}/N \cong S_3.$$

   Feel free to appeal to your geometric intuition. (In light of this fact, you may wish to compare your multiplication table for D_{12}/N with your multiplication table for S_3 from Homework 3.)