1. (For those of you who didn’t look this up yet.) Let $R$ be a commutative ring and let $\langle M_i \rangle$ be the set of maximal ideals of $R$. Prove:

$$\bigcup_i M_i = R \setminus R^*.$$  

2. Let $R = \mathbb{Z}[X]$ and $I = (2, X)$. Observe that $I$ is maximal. Find an $I$-primary ideal that is not a power of $I$.

3. Let $R$ be a ring such that each local ring $R_p$ is Noetherian. Is $R$ Noetherian? Give a proof or counterexample.

4. Let $f : R \to R'$ be a ring homomorphism, and let $S$ be a multiplicative subset of $R$. Prove that $S^{-1}R'$ and $f(S)^{-1}R'$ are isomorphic as $S^{-1}R$ modules.

5. Let $R$ be a ring such that each local ring $R_p$ is an integral domain. Is $R$ an integral domain? Give a proof or counterexample.

6. Let $R$ be a ring with the property that every ideal $I$ is decomposable. Show that $S^{-1}R$ has the same property (for every $S$).

7. Let $A$ be a ring, and $A \subseteq B$ an integral extension ring. Prove that we have $\dim(A) = \dim(B)$.

8. Let $A$ be a domain which is integrally closed in its field of fractions $K$. Let $L/K$ be a normal field extension of $K$ (in the sense of Galois theory). Let $B$ be the integral closure of $A$ in $L$. Prove that all prime ideals of $B$ lying over a prime ideal $p$ of $A$ are conjugate over $K$. (The case $[L : K] < \infty$ will be done in class, your mission is to do the infinite case.)