

1. Find a least-squares solution \hat{x} to the equation $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } b = [1 \ 3 \ 8 \ 2]^T.$$

Is \hat{x} unique? If yes, prove it. If no, give a description of *all* least-squares solutions to $Ax = b$.

2. Let

$$A = \begin{bmatrix} 1 & 5 \\ 3 & 1 \\ -2 & 4 \end{bmatrix} \text{ and } b = [4 \ -2 \ -3]^T.$$

Compute the projections of b onto $\text{Col}(A)$. Is it possible to compute the projection of b onto $\text{Row}(A)$?

3. Section 6.7, exercise 1 from the book (page 300).

4. Let $A = \begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & 2 \\ -2 & 2 & 2 \end{bmatrix}$. Verify that $v_1 = [-2 \ 2 \ 1]^T$ and $v_2 = [1 \ 1 \ 0]^T$ are eigenvectors of A . Orthogonally diagonalize A .

5. Construct a spectral decomposition for the matrix A of exercise 4.