A Taste of the Infinite: Syllabus

Rich Schwartz

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Course Title: *A Taste of the Infinite*

Instructor: Prof. Rich Schwartz

Time/Place: Tu.-Th. 2:30-3:50, in Salomon 004

Course Description: *A Taste of the Infinite* is a freshman seminar designed to give students a feel for some of the many ways that the infinite enters into higher mathematics. The main idea is to expose students to things in mathematics that they wouldn't otherwise get to see unless they went far into the math concentration.

The class will have a number of modules which introduce topics in various areas of mathematics. There are probably more topics below than I can cover in a semester, but I'll try to get through as much as I can. Roughly, there will be one topic per week.

Course Assessment: The grade for the course will be based on weekly homework assignments and a final project. The weekly assignments will usually be tied pretty closely to the course material. The final projects, on the other hand, are meant to be fairly open-ended.

Text: There isn't a textbook for the class, though sometimes I will give handouts or other kinds of readings.

Topics:

1. Big Finite Numbers. This will be a discussion of big finite numbers and the basic arithmetic operations (like plus and times).
2. Cardinality: Integers, Rationals, Binary Sequences: Cantor's diagonalization argument.
3. Construction of the real numbers.
4. Complex numbers and quaternions. Basic definition and properties of complex numbers and quaternions.
5. P-adic numbers. Definition of the p-adic numbers and some of their basic properties.
6. groups: definition of a group, and a lot of examples. The examples will concentrate on infinite groups.
7. Cayley graphs. How to associate a graph to a finitely generated groups; examples – Free group, affine group, Nil, Sol,...
8. limits: limits of numbers, Hausdorff topology, geometric limits.
9. fractals: Cantor set, Sierpinski triangle, Sierpinski Carpet, Menger Sponge, Peano curve.
10. Dimension: Hausdorff dimension computed for some simple examples.
11. Non-euclidean geometry: projective geometry, hyperbolic geometry.
12. Higher dimensional space: Euclidean space, (maybe) Hilbert space and the Hilbert cube.
13. The Axiom of choice: Zorn's lemma, ultrafilters, intro to the hyperreals.