Math 123 HW 6

1. Prove the following result. For each integer $n > 0$ there is some other N with the following property. If the edges of the complete bipartite graph $K_{N,N}$ are colored red and blue then there is some monochromatic copy of $K_{n,n}$ inside the colored graph.

2. This is problem 8.3.10 from the book: Prove that every 9 point subset of the plane, with no three points collinear, contains a convex pentagon. Exhibit a set of 8 points in the plane (with no three collinear) that does not have this property.

3. This is part of Problem 8.3.13 in the book. For each integer $k \geq 2$ prove that there is some K with the following property. If the integers $1, 2, ..., K$ are colored with k colors then there exists 3 points a, b, c such that $a + b = c$ and a, b, c all get the same color. Hint: turn this into a problem about graphs and use the Ramsey Theorem.

4. This is problem 5.1.22 in the book. Given a finite set of lines in the plane, with no three meeting at a point, form a graph whose vertices are the intersection points and whose edges join vertices on the same line. Prove that this graph is 3-colorable.

5. Suppose that each point in the plane is colored one of 3 colors. Prove that there exists two points, one unit apart, which have the same color.

6. Show that the conclusion in Problem 5 is false is you use 7 colors. Hint: Try some kind of tiling of the plane. Nobody knows whether the result is true or false for $4, 5, 6$ colors.