Math 20 Midterm 1. 5 Mar 2008
Instructions. Show all your work.

1. Consider the function

$$f(x, y, z) = x^2z - 3y^2 + 4xz^2 - 2.$$

Note that $f(1, 1, 1) = 0$. Write the equation for the plane tangent to the level surface $f = 0$ at the point $(1, 1, 1)$.

2. Let $f(x, y, z) = \frac{1}{3}x^3 + xyz$. Let $D_v f$ stand for the directional derivative of f in the direction of v. Let $p = (1, 2, 2)$.
a. Find a unit vector v such that $D_v f(p) = 5$.
b. Find a unit vector v such that $D_v(f)(p) = 0$ and $v \cdot (0, -1, 1) = 0$.
c. Explain why there is no unit vector v such that $D_v f(p) = 6$.
d. Explain why there are infinitely many unit vectors v such that $D_v f(p) = 4$.

3. Consider the function $f(x, y) = xy + x$ defined on the disk D given by $x^2 + y^2 \leq 3$. Find and classify all the critical points of f on D, and also find the values where f attains its minimum and its maximum.

4. The functions $r_1(t)$ and $r_2(t)$ describe the positions of two particles in the plane. The first particle moves along the x-axis at speed 2 in such a way that $r_1(0) = (1, 0)$. The second particle moves in such a way that $r_2(0) = (2, 3)$ and $r_2'(0) = (1, 1)$. Let $E(t)$ denote the square of the distance between $r_1(t)$ and $r_2(t)$. For instance

$$E(0) = (2 - 1)^2 + (3 - 0)^2 = 10.$$

Use the chain rule to compute

$$\frac{dE}{dt}(0).$$

(Note, even though this problem involves particles in the plane, the function E is a function of more than 2 variables, because it depends on the coordinates of both points.)