

Guide to the Monograph:

My monograph is mostly about proving a general theorem about spherical CR structures and Dehn surgery. This result has a variety of corollaries, including a resolution of the “triangle groups conjecture” for all small-angle complex hyperbolic reflection triangle groups. The purpose of these notes is to guide you through what you need to know in order to understand the last ideal triangle group – the most interesting one of all.

Preliminaries: Here are some preliminaries.

- For a definition of complex reflections, see Equation 2.6.
- For basics on Heisenberg space, see Section 2.3. However, it might be better to read the notes I already posted on Heisenberg space.
- To see the box product and the angular invariant, read Sections 2.5.1 and 2.5.2.
- Section 4.4 talks about triangle groups in general, and Section 4.6 defines the ideal triangle groups. You can read 4.6 without knowing 4.4.

Parabolic R -cones Recall from class that the proof that the last ideal triangle group is discrete amounts to constructing 3 special spheres. Sections 19.1 and 19.2 have the basic construction. Section 19.3 discusses the elevation (i.e. cylindrical) coordinate system used in class. Section 19.4 contains some technical details about these spheres which I swept under the rug. These details are needed to prove those statements I mentioned concerning the monotonicity of the arcs involved.

Construction of the spheres: Section 20.1 gives the main construction, which I gave in class. Section 20.2 gives the disjointness proof, modulo some details. Section 20.3 deals with a property of this group which is irrelevant for just the discreteness proof. You can ignore it. The details of the disjointness proof you need to know are in Sections 20.4 and 20.5. You can also ignore Section 20.6.

Analysis of the Manifold at Infinity: Section 21.1 builds the model of the fundamental domain which I described in class. Section 21.2 builds the abstract simplicial complex. Section 21.3 analyzes the ideal triangle group

acting on the simplicial complex and shows the the quotient is the Whitehead link complement. Section 21.4 identifies the abstract model with the domain of discontinuity for the triangle group.

The Rest of the triangle Groups Sections 22.1 and 22.2 generalize the construction in Section 20.1 to the case of groups near the critical value. Once you read these sections, Figures 22.1 and 22.2 will probably make sense. These are picture proofs that the “3-ball construction” still works in the loxodromic case.