Math. 0540-01 Fall 2008. Test 1.

Name:
Student #

Problem	1	2	3	4	5	6	7	total
Max Possible	14	14	14	14	14	15	15	100
Score								

Work out the problems on the space provided. If more space is needed use back side of the pages or scratch paper.

To ensure maximal partial credit show all your work.

After you are done with the test, please comment on its difficulty:
The test was:
☐ Too easy ☐ Fairly easy ☐ Right on ☐ Challenging ☐ Unfairly demanding

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1. True or false: If A is 9×14 matrix with 5-dimensional nullspace $\operatorname{Ker} A$, then $A\mathbf{x} = \mathbf{b}$ can be solved for every $\mathbf{b}$. Justify

2. Let A and B be $n \times n$ matrices. True or false: if the product AB is invertible, then both A and B are invertible. Justify.

3. An $n \times n$ matrix A is called *skew symmetric* if $A^T = -A$. Prove that if A is skew symmetric and n is odd, then $\det A = 0$.

4. For the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & 2 & 8 & 3 \end{pmatrix}$$

find its rank and the dimensions of four fundamental subspaces.

Find bases in its column space $\operatorname{Ran} A$, row space $\operatorname{Ran} A^T$ and in the nullspace $\operatorname{Ker} A$.

5. For what value of b the system

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 6 & 3 \\ 3 & 9 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ 7 \\ b \end{pmatrix}$$

has a solution. Find the general solution of the system for this value of b.

Do ether this problem, of Problem 6 alt. on the next page. Do **only one** of these two problems.

6. Let a linear transformation $T : M_{2\times 2} \to M_{2\times 2}$ is given by the formula $T(X) = AX$, $X \in M_{2\times 2}$, where

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Find the matrix of T in the basis

$$\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}.$$

6 alt. A linear transformation in $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5x + 2y \\ 3y - 2x \end{pmatrix}$$

Find the matrix of this transformation in the standard basis and in the basis $(1, 3)^T$, $(-3, 1)^T$.

7. Find the matrix of the reflection through the line $y = 3x$.