

Math 1620 Homework 8

Due April 14

1. We have a single sample $S = \{X_1\}$ from a Poisson-distributed RV with unknown mean λ . Find an unbiased estimator of $e^{-\lambda}$. What does the CR inequality tell us about an unbiased estimator of $e^{-\lambda}$? Is there a better biased estimator?
2. Let S be a sample of size n from an exponential distribution of unknown rate λ (that is, the distribution has density $f(x) = \lambda e^{-\lambda x}$). Let Y be the sample mean; show that $Y \log 2$ is an unbiased estimator of the median. Compare its variance with the CR bound.
3. Find the density of the largest element in a sample of n points from an exponential distribution with parameter λ .
4. Given a sample of size $2m + 2$ from a normal distribution, what is the distribution of the joint RV (Y_{m+1}, Y_{m+2}) ? Show that the difference $Y_{m+2} - Y_{m+1}$ is approximately exponential, that is, has density converging to that of an exponential RV as $m \rightarrow \infty$.