

Math 1620 Homework 9

Due April 21

1. The *Laplace distribution* with mean μ is the probability measure on \mathbb{R} with density $f(x) = \frac{1}{2}e^{-|x-\mu|}$. Show that the sample median is a better estimator of the population mean (in the sense of having lower MSE) than the sample mean.
2. X is a two-dimensional normal RV with density $f(x, y) = Ce^{-(x^2+xy+y^2)}$. What is the covariance matrix? Compute $\mathbb{E}(x^i y^j)$ for $0 \leq i, j \leq 2$.
3. S is a sample of size n from a uniform distribution on an interval $[\theta, \theta + 1]$. What is the maximal likelihood estimator for θ ? What is your best guess for θ ? How far is this guess likely to be off from the true value?
4. In the previous problem, suppose that θ is known to be chosen from a standard normal distribution. Suppose $n = 1$ and $S = \{1.0\}$ rounded to the nearest .1. What does the maximal likelihood estimate tell you now about θ ?