Math 1260
final exam

due May 16, 2008, by 5pm, in R. Kenyon's office, Kassar 305
Duration: 3 consecutive hours

Name: _____
Start time and date: _____
Finish time and date: _____
Signed: _____

You may use the class text and your notes but no other texts or information sources. No calculators, computers, web resources. You must complete the exam in one sitting, that is, in one three-hour period. Please write complete sentences. Use English connectors between equations, like "thus" and "which implies" and so on.

1. Points $1+i, 3+2i$ are two vertices of a square. Find the two other vertices in all possible cases.

2. Let $0 < a < 1$. Show that $\phi(z) = \frac{z-a}{1-az}$ maps the unit disk $\{z : |z| < 1\}$ bijectively to itself.

3. Compute

$$\int_{|z|=4} \frac{z^3}{\sin z} dz.$$

4. Show that, under an appropriate choice of branch for $\sqrt{z}$, the function $f(z) = \frac{\sin\sqrt{z}}{\sqrt{z}}$ is analytic on $\mathbb{C} \setminus \{0\}$ with an isolated singularity at 0. What kind of isolated singularity does it have? Compute the Laurent expansion about $z = 0$.

5. Compute $\int_{|z|=4} \tan z \, dz$. Let $\tan z = f_0(z) + f_1(z)$ be the Laurent decomposition of $\tan z$ on the annulus $2 < |z| < 4$. What is $f_1(z)$?

6. For what values of $z \in \mathbb{C}$ is

$$\sum_{n=0}^{\infty} \left(\frac{1-iz}{i-z}\right)^n$$

convergent? Absolutely convergent?

7. Show that the series $\sum_{n\in\mathbb{Z}} e^{-n^2+z/n}$ converges and represents an entire function of z.

8. Compute

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2+x+1} dx.$$