

Homework 2

January 30, 2008

page 11, 1. Express all values of the following in both Cartesian and polar coordinates, and plot.

(b) $(-1)^{1/4}$

(e) $(-8)^{1/3}$

page 11, 6. Fix $n \geq 1$ Show that the n th roots of unity $\omega_0, \dots, \omega_{n-1}$ satisfy

(a) $(z - \omega_0) \dots (z - \omega_{n-1}) = z^n - 1.$

(b) $\omega_0 + \dots + \omega_{n-1} = 0$ if $n \geq 2.$

(c) $\omega_0 \dots \omega_{n-1} = (-1)^{n-1}$

(d) $\sum_{j=0}^{n-1} \omega_j^k = \begin{cases} 0, & 1 \leq k \leq n-1 \\ n & k = n. \end{cases}$

page 13, 1. Sketch the image under stereographic projection of the following sets on the sphere: (a) the lower hemisphere $Z \leq 0.$ (b) the polar cap $\frac{3}{4} \leq Z \leq 1.$

page 13, 2. If the point P on the sphere corresponds to z under S.P., show that the antipodal point $-P$ on the sphere corresponds to $-1/\bar{z}.$

page 13, 4. Show that a rotation of the sphere of 180° about the X -axis corresponds under S.P. to the inversion $z \mapsto 1/z$ of $\mathbb{C}.$

page 18, 1. Sketch the curve and its image under $z \mapsto z^2:$

(a) $|z - 1| = 1$

(d) $y = x + 1$

(e) $y^2 = x^2 - 1, \quad x > 0$

page 18, 2. Sketch the images of the curves in the preceding problem under the principal branch of $\sqrt{z}.$ Sketch on the same grid but in a different color the image under the other branch.

page 21, 1(e) Calculate and plot e^z for $z = \pi i/m, \quad m = 1, 2, 3, \dots$

page 21, 2(b) Sketch the set $\{5\pi/3 < \Im z < 8\pi/3\}$ and its image under $e^z.$ Indicate images of horizontal and vertical lines.

page 24, 1. Find and plot $\log z$ for $z = i$ and $z = 1 + i.$ Specify the principal values.

page 24, 3. Define explicitly a continuous branch of $\log z$ in the complex plane slit along the negative imaginary axis, $\mathbb{C} \setminus [0, -i\infty).$

page 27, 1. Find all values and plot: $(1 + i)^i$ and $2^{-1/2}.$