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Math 0100: Introductory Calculus II
Practice Final

What follows is what I believe to be a reasonable practice final, a cumulative exam with a substantial slant toward more recent material. As was the case last time, I have not seen any draft of the actual final before constructing this, and so the relative ease or difficulty of this test should not necessarily be an indicator of the relative ease or difficulty of the actual midterm. It should also be noted that this exam is far from comprehensive: there is a lot of material you need to know for the final, much of which isn't covered by this practice test. This practice exam needs to be used along with, and not in place of, review to be an effective study tool.

I advise that you study material before trying this practice exam, and then take this under timed, test-conditions (no book, no calculator, 3 hours).

1) Determine whether

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

is absolutely convergent, conditionally convergent or divergent. Justify your answer.

2) Express

$$\frac{1}{1+27x^3}$$

as a Maclaurin series, and give the interval of convergence.

3) Use either multiplication or division of power series to give the first three nonzero terms in the Maclaurin series for the function

$$\frac{e^x}{1-x}.$$

4) Find the centroid of the region bounded by $y = 2x$, $y = x/2$ and $x = 2$.

5) Let a be a constant such that $-1 < a < 0$. Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{a^n(n+a)^2}$$

in terms of a.

6) Evaluate the indefinite integral as a power series. What is the radius of convergence?

$$\int \frac{1}{t^2}\left(\frac{1}{1-t^4}-1\right)\, dt.$$

7) Evaluate $\int x^3\sqrt{4-x^2}\, dx$.

8) Find the implicit solution to the differential equation that satisfies the given initial condition:

$$y^2\frac{dy}{dx}=(x^2+6)e^y, \qquad y(1)=0$$

9) Evaluate

$$\int \frac{x^3+7x^2+20x+28}{(x+2)^2(x^2+4)}\, dx$$

10) Give the Taylor series expansion of $f(x)=e^x$ centered at $a=1$. (Don't worry about $R_n(x)$ or the radius of convergence.)