

MATH0100: SUMMARY NOTE FOR MIDTERM#2

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This is a summary note for midterm #2. I divided odd exercise problems in two groups: we can solve Level 1 just using the material in each section, but for Level 2, we need what we learned before. Hints are in the last section. A practice exam and its solution are on the web(<http://www.math.brown.edu/~tahulse/ma0100.html>).

1. APPROXIMATE INTEGRATION(7.7)

Review homework problems and how to use formulas in this section: Midpoint Rule(p.496), Trapezoidal Rule(p.497), Simpson's Rule(p.502) and Error Bounds(p.499, 503).

2. IMPROPER INTEGRALS(7.8)

Improper integrals are not too much different from usual integrals. The only difference is taking limit at last. Sometimes, taking limit is confusing, for example, if the limit is $(\frac{0}{0})$ or $(\frac{\infty}{\infty})$. In these cases, we can use l'Hospital's rule. For example, in Exercise #40, we did

$$\lim_{x \rightarrow 0} s \ln s = \lim_{x \rightarrow 0} \frac{\ln s}{1/s} = \lim_{x \rightarrow 0} \frac{(\ln s)'}{(1/s)'} = \lim_{x \rightarrow 0} \frac{1/s}{-1/s^2} = \lim_{x \rightarrow 0} (-s) = 0.$$

Practice odd problems:

Level 1: #5, #7, #9, #15, #27, #29, #33, #42, #46.

Level 2: #11, #13, #17, #19, #21, #23, #25, #35, #39, #44.

3. ARC LENGTH(8.1)

The arc length formula: If a curve is a function of x , i.e., $y = f(x)$, and f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leq x \leq b$, is

$$L = \int_a^b \sqrt{a + [f'(x)]^2} dx = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx.$$

One the other hand, if a curve is a function of y , i.e., $x = g(y)$, and g' is continuous on $[c, d]$, then the length of the curve $x = g(y)$, $c \leq y \leq d$, is

$$L = \int_c^d \sqrt{a + [g'(y)]^2} dy = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy.$$

Practice odd problems:

Level 1: #1, #3, #5, #7.

Level 2: #9, #11, #13, #15.

4. APPLICATIONS TO PHYSICS AND ENGINEERING(8.3)

If the region \mathcal{R} lies between $y = f(x)$, $y = g(x)$, where $f(x) \geq g(x)$, then the centroid of \mathcal{R} is (\bar{x}, \bar{y}) where

$$A = \int_a^b [f(x) - g(x)]dx,$$

$$\bar{x} = \frac{1}{A} \int_a^b x[f(x) - g(x)]dx, \quad \bar{y} = \frac{1}{A} \int_a^b \frac{1}{2}[f(x)^2 - g(x)^2]dx.$$

Before applying the above formula, draw a picture first. It is sometimes very helpful. For example, the picture of #25 is symmetric about y-axis. So, $\bar{x} = 0$, directly from the picture without any computation. The picture of #33 is symmetric about x-axis. So, $\bar{y} = 0$. The picture of #29 is symmetric about the graph $y = x$. So, $\bar{x} = \bar{y}$, hence it suffices to compute one of \bar{x} and \bar{y} .

Practice odd problems:

Level 1: #25, #27, #29, #31, #33.

5. SEPARABLE EQUATIONS(9.3)

First, let's recall a standard example(Example 1(p.581)):

$$\frac{dy}{dx} = \frac{x^2}{y^2}; \quad y(0) = 2.$$

$$\begin{aligned} \frac{dy}{dx} = \frac{x^2}{y^2} &\Rightarrow (\text{Separate variables}) \quad y^2 dy = x^2 dx \\ &\Rightarrow (\text{Integrate both sides}) \quad y^3 = x^3 + C \\ &\Rightarrow (\text{Find } C \text{ using } y(0)=2) \quad 8 = y(0)^3 = 0^3 + C = C; \quad C = 8 \\ &\Rightarrow (\text{Get the solution}) \quad y = (x^3 + C)^{\frac{1}{3}} = (x^3 + 8)^{\frac{1}{3}}. \end{aligned}$$

Practice odd problems:

Level 1: #1, #3, #5, #11.

Level 2: #7, #9, #13, #15, #17.

6. HINTS FOR ODD PROBLEMS

(p.516)

#11: substitute $s = 1 + x^2$.

#13: substitute $s = x^2$.

#17: substitute $s = x^2 + 2x$.

#19: integration by part.

#21: substitute $s = \ln x$

#23: substitute $s = x^3$.

#25: substitute $s = \ln x$.

#35: partial fraction.

#39: integration by part(LIATE rule).

#44: substitute $s = x^2 + 9$.

(p.530)

#9: as we did in #10.

#11: as we did in #10.

#13: $(\ln(\sec x))' = \frac{(\sec x)'}{\sec x} = \frac{(\frac{1}{\cos x})'}{\frac{1}{\cos x}} = \frac{(\frac{\sin x}{\cos^2 x})}{\frac{1}{\cos x}} = \frac{\sin x}{\cos x} = \tan x. \Rightarrow \sqrt{1 + (\ln(\sec x))^2} = \sqrt{1 + \tan^2 x} = |\sec x|.$

#15: $\sqrt{1 + (\frac{dy}{dx})^2} = \sqrt{1 + (\frac{-2x}{1-x^2})^2} = \dots = \sqrt{(\frac{1+x^2}{1-x^2})^2} = \frac{1+x^2}{1-x^2} \Rightarrow$ partial fraction.

(p.586)

#7: substitute $s = y^2$.

#9: $2 + 2u + t + tu = (t + 2)(1 + u).$

#13: integration by part for $x \cos x$.

#15: $\int \sec^2 x dx = \tan x$, since $(\tan x)' = \sec^2 x$.

#17: $\int \frac{1}{\tan x} dx = \int \frac{\cos x}{\sin x} = \ln |\sin x| + C$ (the last step follows from the substitution $s = \sin x$).