Mathematics 0100 Practice Final Exam Solution

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Student's Name	1	2	3	4	5	6	7	8	9	10	total

1. Find the radius of convergence and interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{1}{(n+1)} \left(-\frac{x}{2}\right)^n$$

2. Express the function as the sum of power series. Find the radius of convergence.

$$f(x) = \frac{3}{x^2 - x - 2}$$

3. Use power series to evaluate the limit

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$$

4. Determine whether the series is convergent or divergent. If it is convergent, find the sum

$$\sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)} \right)$$

5. Solve the differential equation

$$\frac{dy}{d\theta} = \frac{e^y \sin^2 \theta}{y \sec \theta}$$

6. Find the length of the arc of the parabola $y = e^x$ from (0,1) to (1,e).

7. Determine whether the integral is convergent or divergent

$$\int_0^\infty \frac{x}{(x^2+2)^2} dx$$

8. Evaluate the integral

$$\int_0^1 \frac{1}{1+\sqrt{y}} dy$$

9. Find the values of p for which the following series is convergent

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

10. If $\sum a_n$ is a convergent series with positive terms, is it true that

$$\sum [e^{a_n} - 1]$$

is also convergent?