p. 703 Determine whether the series is convergent or divergent

4. $\sum_{n=1}^{\infty} \frac{1}{n^5}$ $\quad \int_1^{\infty} \frac{1}{x^5}dx = -\frac{1}{4x^4}\Big|_1^{\infty} = 0 + \frac{1}{4} = \frac{1}{4}$. $\therefore$ By Int test, series converges.

6. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$ $\quad \int_1^{\infty} \frac{dx}{\sqrt{x+4}}$ $\quad$ Let $u = x+4$, $du = dx$ $\quad \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$, so

$\int_1^{\infty} \frac{dx}{\sqrt{x+4}} = 2\sqrt{x+4}\Big|_1^{\infty}$, which diverges. $\therefore$ By int test, series diverges.

10. $\sum_{n=1}^{\infty} n^{-1.4} + 3n^{-1.2}$

$\int_1^{\infty} x^{-1.4} + 3x^{-1.2}dx = -\frac{2.5}{x^{0.4}} - \frac{15}{x^{0.2}}\Big|_1^{\infty} = 0 + 2.5 + 15 = 17.5$

$\therefore$ By int test, converges.

14. $\sum_{n=1}^{\infty} \frac{1}{3n+2}$

$\int_1^{\infty} \frac{dx}{3x+2} = \frac{1}{3}\log(3x+2)\Big|_1^{\infty}$, $\log \infty$ diverges

$\therefore$ By int test, diverges

22. $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$

$\int_2^{\infty} \frac{dx}{x(\ln x)^2}$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int \frac{du}{u^2} = -u^{-1} = -\frac{1}{\ln x}\Big|_2^{\infty} = \frac{1}{\ln 2}$

$\therefore$ By int test, converges

41. Find all positive values of b for which the series $\sum_{n=1}^{\infty} b^{\ln n}$ converges

$$b^{\ln n} = e^{(\ln b)^{\ln n}} = e^{\ln n \cdot \ln b} = e^{(\ln n)^{\ln b}} = n^{\ln b},$$

So $\sum_{n=1}^{\infty} b^{\ln n} = \sum_{n=1}^{\infty} n^{\ln b}$. We know that $\sum_{n=1}^{\infty} n^p$ converges as long as $p < -1$.

So, $\ln b < -1$

$\boxed{0 < b < \frac{1}{e}}$ is the interval of convergence of b.

MATH100 Homework Answers

P. 703

42 $\sum_{n=1}^{\infty}\left(\frac{c}{n}-\frac{1}{n+1}\right)$

$$\int_1^{\infty}\frac{c}{x}-\frac{1}{x+1}\,dx = \underbrace{c\log x-\log(x+1)\Big|_1^{\infty}}_{\text{call this } f(x)}$$

If $c=1$, $f(x) = \log x - \log(x+1)\Big|_1^{\infty} = \log 2$ <u>converges</u>

$c>1$ $f(x) = c\log x - \log(x+1)\Big|_1^{\infty}$

$$= \underbrace{\log x - \log(x+1)}_{\text{converges}} + \underbrace{(c-1)\log x}_{\text{diverges, goes to } \infty.}\Big|_1^{\infty}$$

$c<1$ $f(x) = c\log x - \log(x+1)\Big|_1^{\infty}$

$$= \underbrace{\log x - \log(x+1)}_{\text{converges}} + \underbrace{(c-1)\log x}_{\text{diverges, goes to } -\infty.}\Big|_1^{\infty}$$

Series converges when $c=1$.

NOTE: TO USE INTEGRAL TEST YOU MUST FIRST VERIFY THAT THE SERIES IS POSITIVE DECREASING. WHILE THIS IS FUNCTION, $\frac{c}{n}-\frac{1}{n+1}$, SATISFIES THIS WHEN $c>1$, IT IS ACTUALLY EVENTUALLY NEGATIVE INCREASING, SO YOU NEED TO USE THE INTEGRAL TEST ON $-\left(\frac{c}{n}-\frac{1}{n+1}\right)$.

<u>P. 709</u> Determine whether the series converges or diverges

4. $\sum_{n=2}^{\infty}\frac{n^3}{n^4-1}$. Let $b_n=\frac{1}{n}$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\frac{n^3}{n^4-1}}{1/n} = \lim_{n\to\infty}\frac{n^4}{n^4-1} = 1.$$

$\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, so $\sum_{n=2}^{\infty}\frac{n^3}{n^4-1}$ <u>diverges</u>

6. $\sum_{n=1}^{\infty}\frac{n-1}{n^2\sqrt{n}}$ Let $b_n=\frac{1}{n^{1.5}}$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\frac{n-1}{n^2\sqrt{n}}}{1/n^{1.5}} = \lim_{n\to\infty}\frac{n^{2.5}-n^{1.5}}{n^{2.5}} = 1$$

$\sum_{n=1}^{\infty}\frac{1}{n^{1.5}}$ converges, so $\sum_{n=1}^{\infty}\frac{n-1}{n^2\sqrt{n}}$ <u>converges</u>

14. $\sum_{n=2}^{\infty}\frac{\sqrt{n}}{n-1}$ Let $b_n=\frac{1}{\sqrt{n}}$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\frac{\sqrt{n}}{n-1}}{1/\sqrt{n}} = \lim_{n\to\infty}\frac{n}{n-1} = 1$$

$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ diverges $\Rightarrow \sum_{n=2}^{\infty}\frac{\sqrt{n}}{n-1}$ <u>diverges</u>

16. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^3+1}}$ Let $b_n=\frac{1}{n^{3/2}}$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\frac{1}{\sqrt{n^3+1}}}{1/n^{3/2}} = \lim_{n\to\infty}\frac{n^{3/2}}{\sqrt{n^3+1}} = 1$$

$\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ converges $\Rightarrow \sum_{n=1}^{\infty}\frac{1}{\sqrt{n^3+1}}$ <u>conv.</u>

24. $\sum_{n=1}^{\infty}\frac{n^2-5n}{n^3+n+1}$ let $b_n=\frac{1}{n}$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{n^3-5n^2}{n^3+n+1} = 1$$

$\sum_{n=1}^{\infty}\frac{1}{n}$ diverges $\Rightarrow \sum_{n=1}^{\infty}\frac{n^2-5n}{n^3+n+1}$ <u>diverges</u>

26. $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$

$$\frac{n+5}{\sqrt[3]{n^7+n^2}} < \frac{n+5}{n^{7/3}} < \frac{n}{n^{7/3}} = n^{-4/3}.$$

$\sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$ converges, so by comparison,

$\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$ <u>converges</u>.

45. If $\sum a_n$ is a convergent series with positive terms, is it true that $\sum \sin(a_n)$ is also?

$\sum a_n$ converges $\Rightarrow \{a_n\} \to 0$.

$\Rightarrow \{\sin a_n\} \to 0$.

$\lim_{n\to\infty} \frac{\sin a_n}{a_n} = \lim_{x\to 0} \frac{\sin x}{x} = 1$ $\therefore$ Since $\sum a_n$ converges,

$\sum \sin a_n$ <u>converges</u>.

46. If $\sum a_n$ and $\sum b_n$ are convergent with positive terms, is $\sum a_n b_n$ also convergent?

$\sum b_n$ converges $\Rightarrow \{b_n\} \to 0 \Rightarrow$ for all n larger than some M, $b_n < 1$.

So $a_n b_n < a_n$ for all $n > M$

$$\sum_{n=1}^{\infty} a_n b_n = \underbrace{\sum_{i=1}^{M} a_n b_n}_{\text{finite}} + \underbrace{\sum_{M+1}^{\infty} a_n b_n}_{\text{converges by comparison to } a_n.}$$

$\therefore \sum a_n b_n$ is also convergent.