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10a. $\sum_{i=1}^{n} a_i$ { $\sum_{j=1}^{n} a_j$

The expressions are the same; changing the name of the variables

12. $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \ldots$

> $\frac{1}{8}(-2)^{n-1}$ $r = -2$

$|r| = 2 > 1$ > $\boxed{\text{Divergent}}$

14. $1 + 0.4 + 0.16 + 0.064 + \ldots$

$1(0.4)^{n-1}$

$r = 0.4$ $|r| = 0.4 < 1$ $\boxed{\text{Convergent}}$

$$\sum_{n=1}^{\infty} (0.4)^{n-1} = \frac{1}{1-(0.4)} = \frac{1}{0.6} = \frac{5}{3}$$

22. $\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$ $S_n = \sum_{k=1}^{n} \frac{k+1}{2k-3}$

$a_n = \frac{k+1}{2k-3}$ $\lim_{k\to\infty} \frac{k+1}{2k-3} = \lim_{k\to\infty} \frac{1}{2} = \frac{1}{2}$

> Test for convergence: $\boxed{\text{Divergent}}$ b/c $\lim_{k\to\infty} a_n \neq 0$

30. $\sum_{k=1}^{\infty} (\cos 1)^k$ $r = \cos 1 \leq 1$ $a = (\cos 1)^1 = \cos 1$

$|r| = |\cos 1| < 1$ $\boxed{\text{Convergent}}$

$$\sum_{n=1}^{\infty} (\cos 1)^n = \frac{\cos 1}{1-(\cos 1)}$$

38. $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$ $a_n = \ln \frac{n}{n+1}$ > $\lim_{n\to\infty} a_n = \ln\left(\lim_{n\to\infty} \frac{n}{n+1}\right) = \ln\left(\lim_{n\to\infty} \frac{1}{1}\right)$

$= \ln(1) = 0$

{ $\ln \frac{1}{2} + \ln \frac{2}{3} + \ln \frac{3}{4} \ldots$

$(\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + \ldots + (\ln n - \ln(n+1))$

$S_n = \sum_{k=1}^{n} \ln \frac{k}{k+1} = \sum_{k=1}^{n} (\ln k - \ln(k+1)) = -\ln(n+1)$ →

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} -\ln(n+1) = -\lim_{n\to\infty} \ln(n+1) = -\infty$$ $\boxed{\text{Divergent}}$

60. $\sum_{n=0}^{\infty} e^{nc} = 10$ → geometric series

$a = 1$

$r = e^c$

$$\sum_{n=0}^{\infty} e^{nc} = 10 = \frac{1}{1-e^c}$$

$$10(1-e^c) = 1$$

$$1-e^c = \frac{1}{10}$$

$$-e^c = -\frac{9}{10}$$

$$e^c = \frac{9}{10}$$

$$\boxed{c = \ln\left(\frac{9}{10}\right)}$$