Spring 2010
MA 1410, Combinatorial Topology
Handout #1: Isotopy

February 4, 2010

Let's try to make precise the notion of *continuously deforming* one set X to another set Y in, say, $\mathbb{R}^3$. At a minimum, it should mean that sets $X_t \in \mathbb{R}^3$ can be defined for all 'times' t, $0 \leq t \leq 1$, with $X_0 = X$and $Y = X_1$. All the sets X_t should be homeomorphic to each other and in some sense X_t should depend continuously on t. How about requiring that there are maps $f_t : X \to \mathbb{R}^3$ for $0 \leq t \leq 1$ such that (1) for every t f_t is one to one and a homeomorphism onto its image, (2) f_0 is the inclusion map, (3) $f_1(X) = Y$, and (4) the total map $(x, t) \mapsto f_t(x)$ is a continuous map from $X \times I$ to $\mathbb{R}^3$? Or better yet maps $h_t : \mathbb{R}^3 \to \mathbb{R}^3$ for $0 \leq t \leq 1$ such that (1) for every t h_t is a homeomorphism, (2) h_0 is the identity map, (3) $h_1(X) = Y$, and (4) the total map $(x, t) \mapsto f_t(x)$ is a continuous map from $\mathbb{R}^3 \times I$ to $\mathbb{R}^3$.