January 2015
MA 2110, Introduction to Manifolds
First Assignment
Due 2/5/15

January 30, 2015

1. Let $G : U \to V$ and $F : V \to W$ be maps between open subsets of Euclidean spaces, let $p \in U$ be a point, and assume that the derivatives D_pG and $D_{G(p)}F$ exist. Prove the coordinate-free Chain Rule: The derivative of $F \circ G$ at the point p exists and is given by a composition of linear maps:

$$D_p(F \circ G) = D_{G(p)}F \circ D_pG.$$

2. Let $M \subset \mathbb{R}^n$ be a smooth m-dimensional manifold in the sense of the first lecture, or the Supplement #2 (http://www.math.brown.edu/ tomg/211notes.html). Using the definition of the tangent space T_pM given there, use the Chain Rule to obtain two more descriptions of T_pM: First, if a neighborhood of p in M has a regular parametrization $\phi : V \to \mathbb{R}^n$ with $\phi(0) = p$, then T_pM is the image of the linear map $D_0\phi : \mathbb{R}^m \to \mathbb{R}^n$. Second, if a neighborhood of p in M is a regular level set $\psi^{-1}(0)$ then T_pM is the kernel of the linear map $D_p\psi : \mathbb{R}^n \to \mathbb{R}^{n-m}$.

3. For which values of R can we say that the solution set of the pair of equations

$$x^2 + y^2 = 1$$

$$x^2 + z^2 = R^2$$

is a one-dimensional smooth manifold in $\mathbb{R}^3$? Sketch the set for several illustrative values of R.