February 2015
MA 2110, Introduction to Manifolds
Second Assignment
Due 2/12/15

March 1, 2015

4. (a) Prove that a bijection $h : \mathbb{R} \to \mathbb{R}$ is a homeomorphism if and only if it is either order-preserving or order-reversing.
(b) Suppose that M is a topological 1-manifold covered by two charts (U, x) and (V, y), such that $x(U) = \mathbb{R} = y(V)$, and such that $x(U \cap V) = y(U \cap V) = \mathbb{R}_+$, the set of all positive real numbers. Let $h : \mathbb{R}_+ \to \mathbb{R}_+$ be the homeomorphism given by $h(x(p)) = y(p)$ for all $p \in U \cap V$. Show that h must be order-reversing (using the fact that a manifold is required to be a Hausdorff space).
(c) Show that M is homeomorphic to $\mathbb{R}$.
(d) Now assume that h is a diffeomorphism, and note that these two charts then make M a smooth manifold. Show that M is diffeomorphic to $\mathbb{R}$.

[Recall that a polynomial $F(t) = F(t_0, \ldots, t_n)$ is *homogeneous of degree* d if all of the monomials occurring in it have total degree d, that is, it is a sum of terms $ct_0^{d_0} t_1^{d_1} \ldots t_n^{d_n}$ with $d_0 + \ldots + d_n = d$. This means that $F(ut) = u^d F(t)$ for $u \in \mathbb{R}$ and $t \in \mathbb{R}^{n+1}$.]

5. A homogeneous polynomial F as above determines a subset $V(F) \subset \mathbb{R}P^n$, namely the set of all points $(t_0 : \ldots : t_n)$ such that $F(t_0, \ldots, t_n) = 0$. For the example $F(t_0, t_1, t_2, t_3) = t_0^d + t_1^d + t_2^d - t_3^d$, show that $V(F)$ is a smooth 2-manifold in $\mathbb{R}P^3$.