February 2015
MA 2110, Introduction to Manifolds
Third Assignment
Due 2/19/15

March 1, 2015

6. Verify that at any point $(x, y, z) \in S^2$ the vectors $(-y, x, 0)$ and $(xz, yz, z^2 - 1)$ are tangent to S^2. Write these same tangent vectors down in terms of the (stereographic) coordinates $u = \frac{x}{1+z}$, $v = \frac{y}{1+z}$ defined in $S^2 - \{(0, 0, -1)\}$. The answers should have the form $a\frac{\partial}{\partial u} + b\frac{\partial}{\partial v}$ with a and b being functions of u and v.

7. Define $F : \mathbb{R}^2 \to \mathbb{R}^3$ by $F(x, y) = (x, y^2, y^3 - xy)$. Find all points in $\mathbb{R}^2$ at which F fails to be an immersion. Find all points $p \in \mathbb{R}^3$ such that there is more than one point $q \in \mathbb{R}^2$ for which $F(q) = p$. For each such p, determine all of the "tangent planes of $F(\mathbb{R}^2)$ at p" (the images of the linear maps $D_qF : \mathbb{R}^2 \to \mathbb{R}^3$).

8. Show that there is a smooth embedding of $\mathbb{R}P^n$ in the space of real $(n+1)\times(n+1)$ real matrices, defined by $e(x_0 : \ldots : x_n) = \frac{1}{|x|^2}x^t x$. Here $x = (x_0, \ldots, x_n)$ is a nonzero vector in $\mathbb{R}^{n+1}$, considered also as a $1 \times (n + 1)$ matrix, and x^t is the transpose.