February 2015
MA 2110, Introduction to Manifolds
Fourth Assignment
Due 3/5/15

March 1, 2015

9. Verify that the definition of "closed 1-form" given in class is independent of coordinates, in the two-dimensional case. That is, assuming that $\omega = adx + bdy$ satisfies $\frac{\partial a}{\partial y} = \frac{\partial b}{\partial x}$, now suppose that the same cotangent field expressed in different coordinates is $\omega = pdu + qdv$ and show by direct chain-rule calculation that $\frac{\partial p}{\partial v} = \frac{\partial q}{\partial u}$.

10. Let $O(n)$ be the set of all $n \times n$ real matrices A such that $A^t A = \mathbb{I}$. (Note that these are precisely those A such that the corresponding linear operator on $\mathbb{R}^n$ preserves the inner product, since $\langle Av, Aw \rangle = \langle A^t Av, w \rangle$. Thus $O(n)$ is a subgroup of $GL_n(\mathbb{R})$, the group of invertible matrices.) Show that $O(n)$ is a compact smooth submanifold of the vector space of all $n \times n$ matrices, of dimension $\frac{n(n-1)}{2}$.

11. We say that a smooth map $F : M \to N$ is transverse to a smooth submanifold $P \subset N$ if for every $a \in M$ such that $F(a) \in P$ the tangent space $T_{F(a)}N$ is equal to $T_{F(a)}P + (D_a F)(T_a M)$. Note that when F is the inclusion of a submanifold M of N then this is the same as the condition defined in class: M is transverse to P in N. Show that if F is transverse to P then $F^{-1}(P)$ is a smooth submanifold of M.