March 2015
MA 2110, Introduction to Manifolds
Fifth Assignment
Due 3/19/15

March 18, 2015

A *diagonal point* in $M \times M$ is a point (p, q) such that $p = q$. Denote the set of diagonal points by Δ_M. For $F : M \to N$ a *double point* is a point (p, q) such that $p \neq q$ and $F(p) = F(q)$. Thus if we write $F^{(2)} : M \times M - \Delta_M \to N \times N$ for the restriction of the map $F \times F : M \times M \to N \times N$ then the set of double points is the preimage of Δ_N under $F^{(2)}$. Say that the double points of F *occur transversely* if $F^{(2)}$ is transverse to Δ_N. Note that in this case the set of double points is a smooth manifold of dimension $2m - n$ (in particular empty if $2m < n$).

12. Show that the map $F \times F$ can never be transverse to Δ_N at a *diagonal* point of $M \times M$ if $m < n$.

A *non-immersion point* of F is a point $p \in M$ such that the linear map $D_pF : T_pM \to T_{F(p)}N$ is not injective.

13. Prove that if the diagonal point (p, p) is a limit of double points for F then p is a non-immersion point for F.

Let $0_M \subset TM$ be the image of the zero-section $M \to TM$. Denote by $D^{(0)}F$ the restriction of $DF : TM \to TN$ to the complement $TM - 0_M$, so that F is an immersion if and only if the inverse image of 0_N by $D^{(0)}F$ is empty. Say that the non-immersion points of F *occur transversely* if $D^{(0)}F$ is transverse to 0_N.

14. Show that F must be an immersion if the non-immersion points occur transversely and $2m \leq n$.

15. Show that for the map $\mathbb{R}^2 \to \mathbb{R}^3$ of Problem 7 both double points and non-immersion points occur transversely.