## exercises

*(Exercises with colored numbers are solved in the Study Guide.)*

*Complete the computations in Exercises 1 to 4.*

**1.** $(-21, 23) - (?, 6) = (-25, ?)$

**2.** $3(133, -0.33, 0) + (-399, 0.99, 0) = (?, ?, ?)$

**3.** $(8a, -2b, 13c) = (52, 12, 11) + \frac{1}{2}(?, ?, ?)$

**4.** $(2, 3, 5) - 4\mathbf{i} + 3\mathbf{j} = (?, ?, ?)$

*In Exercises 5 to 8, sketch the given vectors* $\mathbf{v}$ *and* $\mathbf{w}$. *On your sketch, draw in* $-\mathbf{v}$, $\mathbf{v}+\mathbf{w}$, *and* $\mathbf{v}-\mathbf{w}$.

**5.** $\mathbf{v} = (2, 1)$ and $\mathbf{w} = (1, 2)$

**6.** $\mathbf{v} = (0, 4)$ and $\mathbf{w} = (2, -1)$

**7.** $\mathbf{v} = (2, 3, -6)$ and $\mathbf{w} = (-1, 1, 1)$

**8.** $\mathbf{v} = (2, 1, 3)$ and $\mathbf{w} = (-2, 0, -1)$

**9.** Let $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j}$. Sketch $\mathbf{v}$, $\mathbf{w}$, $\mathbf{v}+\mathbf{w}$, $2\mathbf{w}$, and $\mathbf{v}-\mathbf{w}$ in the plane.

**10.** Sketch $(1, -2, 3)$ and $(-\frac{1}{3}, \frac{2}{3}, -1)$. Why do these vectors point in opposite directions?

**11.** What restrictions must be made on $x$, $y$, and $z$ so that the triple $(x, y, z)$ will represent a point on the $y$ axis? On the $z$ axis? In the $xz$ plane? In the $yz$ plane?

**12.** (a) Generalize the geometric construction in Figure 1.1.7 to show that if $\mathbf{v}_1 = (x, y, z)$ and $\mathbf{v}_2 = (x', y', z')$, then $\mathbf{v}_1 + \mathbf{v}_2 = (x + x', y + y', z + z')$.

(b) Using an argument based on similar triangles, prove that $\alpha\mathbf{v} = (\alpha x, \alpha y, \alpha z)$ when $\mathbf{v} = (x, y, z)$.

*In Exercises 13 to 19, use set theoretic or vector notation or both to describe the points that lie in the given configurations.*

**13.** The plane spanned by $\mathbf{v}_1 = (2, 7, 0)$ and $\mathbf{v}_2 = (0, 2, 7)$

**14.** The plane spanned by $\mathbf{v}_1 = (3, -1, 1)$ and $\mathbf{v}_2 = (0, 3, 4)$

**15.** The line passing through $(-1, -1, -1)$ in the direction of $\mathbf{j}$

**16.** The line passing through $(0, 2, 1)$ in the direction of $2\mathbf{i} - \mathbf{k}$

**17.** The line passing through $(-1, -1, -1)$ and $(1, -1, 2)$

**18.** The line passing through $(-5, 0, 4)$ and $(6, -3, 2)$

**19.** The parallelogram whose adjacent sides are the vectors $\mathbf{i} + 3\mathbf{k}$ and $-2\mathbf{j}$

**20.** Show that $\mathbf{l}_1(t) = (1, 2, 3) + t(1, 0, -2)$ and $\mathbf{l}_2(t) = (2, 2, 1) + t(-2, 0, 4)$ parametrize the same line.

**21.** Do the points $(2, 3, -4)$, $(2, 1, -1)$, and $(2, 7, -10)$ lie on the same line?

**22.** Let $\mathbf{u} = (1, 2)$, $\mathbf{v} = (-3, 4)$, and $\mathbf{w} = (5, 0)$:

(a) Draw these vectors in $\mathbb{R}^2$.

(b) Find scalars $\lambda_1$ and $\lambda_2$ such that $\mathbf{w} = \lambda_1\mathbf{u} + \lambda_2\mathbf{v}$.

**23.** Suppose $A$, $B$, and $C$ are vertices of a triangle. Find $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.

**24.** Find the points of intersection of the line $x = 3 + 2t$, $y = 7 + 8t$, $z = -2 + t$, that is, $\mathbf{l}(t) = (3 + 2t, 7 + 8t, -2 + t)$, with the coordinate planes.

**25.** Show that there are no points $(x, y, z)$ satisfying $2x - 3y + z - 2 = 0$ and lying on the line $\mathbf{v} = (2, -2, -1) + t(1, 1, 1)$.

**26.** Show that every point on the line $\mathbf{v} = (1, -1, 2) + t(2, 3, 1)$ satisfies the equation $5x - 3y - z - 6 = 0$.

**27.** Determine whether the lines $x = 3t + 2$, $y = t - 1$, $z = 6t + 1$, and $x = 3s - 1$, $y = s - 2$, $z = s$ intersect.

**28.** Do the lines $(x, y, z) = (t + 4, 4t + 5, t - 2)$ and $(x, y, z) = (2s + 3, s + 1, 2s - 3)$ intersect?