LOWER BOUNDS IN THE MATRIX CORONA THEOREM AND THE CODIMENSION ONE CONJECTURE.

SERGEI TREIL

Main result of this paper is the following theorem: given δ, $0 < \delta < 1/3$ and $n \in \mathbb{N}$ there exists an $(n+1) \times n$ inner matrix function $F \in H^\infty_{(n+1)\times n}$ such that

$$I \geq F^*(z)F(z) \geq \delta^2 I \qquad \forall z \in \mathbb{D},$$

but the norm of any left inverse for F is at least $[\delta/(1-\delta)]^{-n} \geq (\frac{3}{2}\delta)^{-n}$. This gives a lower bound for the solution of the Matrix Corona Problem, which is pretty close to the best known upper b bound $C \cdot \delta^{-n-1} \log \delta^{-2n}$ obtained recently by T. Trent. In particular, both estimates grow exponentially in n; the (only) previously known lower bound $C\delta^{-2}\log(\delta^2 n+1)$ (obtained by the author) grew logarithmically in n. Also, the lower bound is obtained for $(n+1) \times n$ matrices, thus giving the negative answer to the so-called "codimension one conjecture." Another important result is Theorem 2.4 connecting left invertiblity in H^∞ and co-analytic orthogonal complements.

Date: December, 2002.

S. Treil is partially supported by an NSF grant.