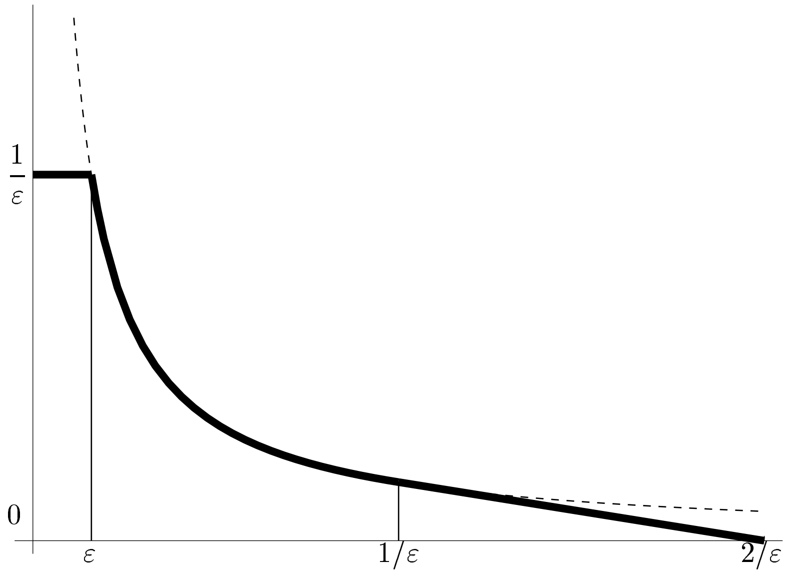

FIGURE 1. One of possible regularizations of the Cauchy kernel. The graph of $|K_\varepsilon(z)|$ vs. $|z|$ is displayed. On the interval $[1/\varepsilon, 2/\varepsilon]$ the function is linear, and the derivatives at the point $1/\varepsilon$ agree

Theorem 1.1. *Let μ be a regular measure, i.e., $\mu(Q) \le \ell(Q)$ for any square $Q \subset \mathbb{C}$. Then the following statements are equivalent*

1. *The Cauchy operator $\mathcal{C} = \mathcal{C}_\mu$ is bounded in $L^2(\mu)$;*
2. *For any square Q, the estimate $\int_{\mathbb{C}} |\mathcal{C}\chi_Q|^2 d\mu \le C\|\chi_Q\|^2_{L^2(\mu)} = C\mu(Q)$ holds;*
3. *For any square Q, the estimate $\int_{2Q} |\mathcal{C}\chi_Q|^2 d\mu \le C'\|\chi_Q\|^2_{L^2(\mu)} = C'\mu(Q)$ holds;*
4. *For any square Q, the estimate $\int_{Q} |\mathcal{C}\chi_Q|^2 d\mu \le C''\|\chi_Q\|^2_{L^2(\mu)} = C''\mu(Q)$ holds.*

Moreover, the norm of $\mathcal{C}$ can be estimated from above by a constant depending only on C (respectively on C' or C'').

Let us notice that the last condition is equivalent to Melnikov's celebrated condition on the local curvature of measure, see [7], [6].

We will prove an even more general theorem about general Calderón–Zygmund operators. Let T be a Calderón–Zygmund operator with kernel