THE MATRIX-VALUED H^p CORONA PROBLEM IN THE DISK AND POLYDISK

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Abstract: In this paper we consider the matrix-valued H^p corona problem in the disk and polydisk. The result for the disk is rather well known, and is usually obtained from the classical Carleson Corona Theorem by linear algebra. Our proof provides a streamlined way of obtaining this result and allows one to get a better estimate on the norm of the solution. In particular, we were able to improve the estimate found in the recent work of T. Trent. Note that, the solution of the H^∞ matrix corona problem in the disk can be easily obtained from the H^2 corona problem either by factorization, or by the Commutant Lifting Theorem. The H^p corona problem in the polydisk was originally solved by K. C. Lin. The solution used Koszul complexes and was rather complicated because one had to consider higher order $\bar\partial$-equations. Our proof is more transparent and it improves upon Lin's result in several ways. First, we deal with the more general matrix corona problem. Second, we were able to show that the norm of the solution is independent of the number of generators. Additionally, we illustrate that the norm of the solution of the H^2 corona problem in the polydisk $\mathbb{D}^n$ grows at most proportionally to $\sqrt{n}$. Our approach is based on one that was originated by M. Andersson. In the disk it essentially depends on Green's Theorem and duality to obtain the estimate. In the polydisk we use Riesz projections to reduce the problem to the disk case.