
Due Friday, 2/16 (collected)

1. I’ll be collecting the problem # 3 on p. 55 from the previous assignment.

2. Prove that if a sequence \( \{a_n\}_{n=1}^{\infty} \) is bounded, and \( \lim_{n \to \infty} b_n = 0 \) then \( \lim_{n \to \infty} a_n b_n = 0. \)

3. Prove that if \( \lim_{n \to \infty} a_n = a \neq 0 \), then \( \lim_{n \to \infty} \frac{1}{a_n} = 1/a. \)

   One of the essential steps here can be the following statement (which you will need to prove if you use it): if \( \lim_{n \to \infty} a_n = a \neq 0 \), then the “tail” of the sequence \( 1/a_n \) is bounded, i.e. there exist numbers \( N \) and \( R \) such that for all \( n > N \) the expression \( 1/a_n \) is defined (i.e. \( a_n \neq 0 \)) and \( |a_n| \leq R. \)

4. State what does it mean that \( \{a_n\}_{n=1}^{\infty} \) is not a Cauchy sequence.

   You cannot start your statement with negation, i.e. you cannot begin “There is no . . . .”.
   Your statement should start with one of the quantifiers, \( \forall \) or \( \exists \).