

## Homework assignment, Oct. 8, 2008.

Discussed next class

1. Let  $A$  be the multiplication by the independent variable  $x$  operator in  $L^2(0,1)$ ,  $Af(x) = xf(x)$ . Show that the range of this operator is not closed. Recall that  $\text{Ran } A = \{Af : f \in D(A)\}$

**Hint:** Consider, for example functions  $g_n = \chi_{[1/n,1)}$ .

2. Prove the theorem about ranges and kernels of the operators

a)  $\text{Ker } A^* = (\text{Ran } A)^\perp$

b)  $\text{Ker } A = (\text{Ran } A^*)^\perp$

c)  $\text{clos Ran } A = (\text{Ker } A^*)^\perp$

d)  $\text{clos Ran } A^* = (\text{Ker } A)^\perp$