Homework assignment, October 5, 2005.

1. A point is randomly thrown in a disc of radius 1. Let $\xi$ be the distance from the point to the center. Assuming that the point is uniformly distributed in the disc, find distribution function, density, mean value and variance of $\xi$.

2. a) Let a point is thrown at random in the disc $x^2 + y^2 \leq 1$, and let $\xi_1, \xi_2$ be $x$ and $y$ coordinates of the point. Assuming that the point is uniformly distributed in the disc, show that the correlation coefficient of $\xi_1, \xi_2$ is zero, but $\xi_1$ and $\xi_2$ are not independent.

b) Assume now that the point is uniformly distributed in the square $-1 \leq x \leq 1, -1 \leq y \leq 1$, and $\xi_1, \xi_2$ are its $x$ and $y$ coordinates respectively. Are $\xi_1$ and $\xi_2$ independent?

3. Let random variables $\xi_1, \xi_2$ satisfy $P\{\xi_1 \leq a, \xi_2 \leq b\} = P\{\xi_1 \leq a\}P\{\xi_2 \leq b\}$ for all $a$ and $b$. Show that $\xi_1$ and $\xi_2$ are independent, i.e. that

$$P\{a_1 < \xi_1 \leq b_1, a_2 < \xi_2 \leq b_2\} = \{a_1 < \xi_1 \leq b_1\}P\{a_2 < \xi_2 \leq b_2\}$$

for all $a_1, a_2, b_1, b_2$.

4. Ch. 4, # 16, 17.

5. St. Petersburg paradox. (Correct version) A person tosses a fair coin until a tail appears for the first time. If the tail appears on the $n$th flip, the person wins $2^n$ dollars. Let $X$ denote the player’s winnings. Show that $E X = +\infty$. 