1. Draw the graphs for Markov chains with transition matrices

   a) \[
   \begin{pmatrix}
   1 - 2p & 2p & 0 \\
   p & 1 - 2p & p \\
   0 & 2p & 1 - 2p \\
   \end{pmatrix}
   \]

   b) \[
   \begin{pmatrix}
   0 & p & 0 & 1 - p \\
   1 - p & 0 & p & 0 \\
   0 & 1 - p & 0 & p \\
   p & 0 & 1 - p & 0 \\
   \end{pmatrix}
   \]

   c) \[
   \begin{pmatrix}
   1/2 & 1/2 & 0 & 0 & 0 & 0 \\
   1/4 & 3/4 & 0 & 0 & 0 & 0 \\
   1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 \\
   1/4 & 0 & 1/4 & 1/4 & 0 & 1/4 \\
   0 & 0 & 0 & 1/2 & 1/2 & 0 \\
   0 & 0 & 0 & 1/2 & 1/2 & 0 \\
   \end{pmatrix}
   \]

   Use the graphs to find persistent and transient states.

2. Consider random walks in dimensions 2 and 3. Suppose the probabilities of going in each possible direction are equal \(1/4\) in dimension 2 and \(1/8\) in dimension 3. Prove that all states in dimension 2 are persistent, while all states in dimension 3 are transient.

   **Hint:** Each coordinate of the random walk in higher dimension is a one-dimensional random walk considered in Example 3 \((p = q = 1/2)\) and the coordinates of the walk are independent. You can use this and Example 3 to compute the transition probabilities \(p_{ii}(n)\).

3. Ch. 7 # 7, 8, 9, 11, 14.

4. For Markov chains in Problem 1 a) and b) determine if limiting probabilities exist (check if the conditions of Theorem 7.4 are satisfied)