# Homework assignment, March 4, 2009.

Due Wednesday, March 11.

1. # 17 p. 46 of the textbook

2. Let $v$ be a subharmonic function in $\Omega$ having a harmonic majorant. Show that function

$$\begin{aligned} u(z) &= \inf\{f(z) : -f \text{ is subharmonic in } \Omega \text{ and } f \geq v\} \\ &= -\sup\{g : g \text{ is subharmonic in } \Omega \text{ and } g \leq -v\} \end{aligned}$$

is harmonic.

This statement would follow from the following

*Proposition. Let $\mathfrak{F}$ be a family of subharmonic functions in a domain $\Omega$ such that*

*a) If $u, v$ lie in $\mathfrak{F}$, then $w$ defined by $w(z) = max(u(z), v(z))$ also lies in $\mathfrak{F}$.*

*b) If $\{z \in \mathbb{C} : |z - z_0| \leq r\} \subset \Omega$ and $u \in \mathfrak{F}$, then the function $v$ defined by*

$$v(z) = \begin{cases} u(z), & |z - z_0| \geq r \\ \textit{Poisson extensions of } u|_{|z-z_0|=r}, & |z - z_o| < r \end{cases}$$

*also belongs to $\mathfrak{F}$.*

*If $w(z) = \sup\{v(z) : f \in \mathfrak{F}\} < \infty$ for all $z$, then $w$ is harmonic in $\Omega$.*

You will need to show that your family of the functions satisfies assumptions of the proposition (which is quite easy), and to prove the proposition.

3. Give an example of a subharmonic function in $\mathbb{D}$ without harmonic majorant.

4. # 2 p. 305.

5. Interpolation in $H^2$: Let $\{z_k\}k$ be a sequence of points in $\mathbb{D}$. Show that the interpolation problem

$$(1 - |z_k|^2)^{1/2} f(z_k) = w_k, \qquad \forall k$$

has a solution $f \in H^2$ for any sequences $\{w_k\}_k \in \ell^2$ if and only if the sequence $\{z_k\}_k$ is interpolating.