MATH 251 PROBLEMS

(1) Consider a partially ordered set \( X \) and let \( \text{Cat}(X) \) be the associated categories (a unique arrow \( x \rightarrow y \) for each pair \( x \leq y \)). Show that the product of \( x \) and \( y \) in \( \text{Cat}(X) \), if exists, is the greatest lower bound of \( x, y \). Identify similarly the coproduct.

(2) Use the previous exercise to cook up a category where products and coproducts don’t always exist.

(3) Let \( Y \) be a set and \( P(Y) \) be the set of all subsets of \( Y \), partially ordered by inclusion. Identify explicitly products and coproducts in \( \text{Cat}(P(Y)) \).

(4) Let \( A \rightarrow B \) be an abelian group homomorphism. What is the fibered product \( A \times_B 0 \) in elementary terms? What is the cofibered coproduct \( 0 \cap^A B \)?

(5) If you have not done so, prove that a group object in \( \text{Groups} \) is an abelian group.

(6) Lang p 115 ex 1,3,4

(7) If \( S \subset R \) contains no zero divisors, show that \( R \rightarrow S^{-1}R \) is injective.

(8) Prove that if \( p \neq q \) are distinct primes, then \( \mathbb{Z}_{(p)} \ncong \mathbb{Z}_{(q)} \).

(9) Let \( M \) be a finitely generated \( R \) module and \( S \subset R \) multiplicative. Show that \( S^{-1}M = 0 \) if and only if there is \( d \in S \) with \( dM = 0 \).

(10) Lang p. 253 ex 2,3,5,6,11

(11) Determine the minimal polynomial of \( \sqrt{2} + \sqrt{3} \)