1. Let \( z = f(x), x = h(t), z = p(t) = f(h(t)) \). Assume \( h(0) = 1, h'(0) = 2, h''(0) = 3 \), 
\( f(1) = 5, f'(1) = 6, f''(1) = 7 \). Calculate \( p(0), p'(0), p''(0) \).

2. Let \( z = f(x, y), x = u(s, t), y = v(s, t), z = p(s, t) = f(u(s, t), v(s, t)) \).

(a) Assume

\[
\begin{align*}
  u(0, 0) &= 1, & v(0, 0) &= 2, \\
  \frac{\partial u}{\partial s}(0, 0) &= 3, & \frac{\partial u}{\partial t}(0, 0) &= 4, \\
  \frac{\partial v}{\partial s}(0, 0) &= 5, & \frac{\partial v}{\partial t}(0, 0) &= 6, \\
  \frac{\partial f}{\partial x}(1, 2) &= -1, & \frac{\partial f}{\partial y}(1, 2) &= -2.
\end{align*}
\]

Find

\[
\frac{\partial p}{\partial s}(0, 0) \quad \text{and} \quad \frac{\partial p}{\partial t}(0, 0).
\]

(b) In addition to the assumptions made in (a), assume that

\[
\begin{align*}
  \frac{\partial^2 u}{\partial s^2}(0, 0) &= 7, & \frac{\partial^2 u}{\partial t^2}(0, 0) &= 8, & \frac{\partial^2 u}{\partial s \partial t}(0, 0) &= 9, \\
  \frac{\partial^2 v}{\partial s^2}(0, 0) &= 10, & \frac{\partial^2 v}{\partial t^2}(0, 0) &= 11, & \frac{\partial^2 v}{\partial s \partial t}(0, 0) &= 12, \\
  \frac{\partial^2 f}{\partial x^2}(1, 2) &= -3, & \frac{\partial^2 f}{\partial y^2}(1, 2) &= -4, & \frac{\partial^2 f}{\partial x \partial y}(1, 2) &= -5.
\end{align*}
\]

Find

\[
\frac{\partial^2 p}{\partial s \partial t}(0, 0).
\]
3. Let \( x, y \) be Cartesian coordinates, and let \( r, \theta \) be polar coordinates. Let \( w = rx \). Find
\[
\frac{\partial w}{\partial r}, \quad \frac{\partial w}{\partial x}, \quad \frac{\partial^2 w}{\partial x \partial r}, \quad \frac{\partial^2 w}{\partial r \partial x}.
\]
Note that the last two are not equal. Why isn’t this a contradiction?

4. Consider the surface \( S \) defined by
\[
xyz + x^3 + y^3 + z^3 = 9,
\]
and the point \( P(1,2,0) \) on \( S \).

(a) We know we can write \( z = z(x, y) \) on \( S \) near \( P \). Calculate
\[
\frac{\partial z}{\partial x}(1,2), \quad \frac{\partial z}{\partial y}(1,2).
\]

(b) Find the tangent plane to \( S \) at \( P \).

5. Let \( R \) be the triangle with corners \((0,1), (1,0), \) and \((3,-1)\). Find the max and min for \( z = f(x,y) = xy - x - y \) on \( R \).

6. Consider the region \( R \) in the \( xy \)-plane bounded by \( y = x^2 \) and \( y = -2x + 3 \). Find the max and min for \( z = f(x,y) = 1 + x + y \) on \( R \).

7. Let \( \mathbf{a} = \langle 1, 2, 3 \rangle, \ \mathbf{v} = \langle 2, 1, 2 \rangle \). Find vectors \( \mathbf{u} \) and \( \mathbf{w} \) so that \( \mathbf{a} = \mathbf{u} + \mathbf{w} \) with \( \mathbf{u} \) parallel to \( \mathbf{v} \) and \( \mathbf{w} \perp \mathbf{v} \).

8. Consider the a parametric curve \( \mathbf{r}(t) \) so that
\[
\mathbf{r}'(0) = \langle 1, 1, 2 \rangle, \quad \mathbf{r}''(0) = \langle 3, 1, -5 \rangle.
\]

(a) At time \( t = 0 \), find \( \mathbf{T} \) and \( \mathbf{N} \).

(b) At time \( t = 0 \), find \( a_T \) and \( a_N \).

(c) At time \( t = 0 \), find the curvature.
9. Consider the parametric curve

\[ \mathbf{r}(t) = \langle 4, 3, 1 \rangle + t \langle 1, 1, 2 \rangle + \frac{1}{2} t^2 \langle 3, 1, -5 \rangle + t^3 \langle -1, 2, 1 \rangle + t^4 \langle 9, 2, 3 \rangle. \]

*Hint: This problem may be related to the previous problem.*

(a) At time \( t = 0 \), find \( \mathbf{v} \) and \( \mathbf{a} \).

(b) At time \( t = 0 \), find \( \mathbf{T} \) and \( \mathbf{N} \).

(c) At time \( t = 0 \), find \( a_T \) and \( a_N \).

(d) At time \( t = 0 \), find the curvature.

10. Let \( \mathbf{a} = \langle 1, 2, 1 \rangle \) and \( \mathbf{b} = \langle 4, 1, 1 \rangle \). Find a vector \( \mathbf{c} \) so that \( |\mathbf{c}| = 2 \) and \( \mathbf{c} \perp \mathbf{a} \) and \( \mathbf{c} \perp \mathbf{b} \).

11. Consider the plane \( x + 2y + 3z = 4 \) and the point \( Q(4, 1, 1) \)

(a) Find the equation of the line \( L \) through \( Q \) which is perpendicular to the plane.

(b) Find the point \( P \) where the line \( L \) meets the plane.

12. Find the equation of the line of intersection of the planes

\[ x + y + z = 1 \quad \text{and} \quad 2x + 3y + 4z = 5. \]

13. Consider points \( A(1, 2, 3), B(2, 1, 1), \) and \( C(4, 3, 1) \).

(a) Find the area of triangle \( ABC \).

(b) Find the equation for the plane passing through \( A, B, \) and \( C \).

14. This problem discussed information about curves at a particular time \( t = t_0 \). It asks you to assume certain information and to calculate other information. Answer “insufficient” if a particular calculation cannot be done.

(a) Assume \( a_T = 2 \) and \( a_N = 3 \). Find \( |\mathbf{a}| \).
(b) Assume that $\mathbf{v} = \langle 1, 1, 1 \rangle$ and $\mathbf{a} = \langle 2, -1, 0 \rangle$. Find $T$, $N$, $\kappa$.

(c) Assume that $\mathbf{v} = \langle 1, 1, 1 \rangle$ and $\mathbf{a} = \langle 2, -1, 0 \rangle$. Find $a_T$, $a_N$, $dT/ds$.

(d) Assume $a_T = 1$, $a_N = 2$, $T = (1/\sqrt{3})\langle 1, 1, 1 \rangle$, $N = (1/\sqrt{2})\langle 1, -1, 0 \rangle$, $\kappa = 3$. Find $\mathbf{v}$ and $\mathbf{a}$.

(e) Assume $dT/ds = \langle 1, 2, 3 \rangle$. Find $\kappa$ and $N$.

(f) Assume $\mathbf{v} = \langle 1, 1, 1 \rangle$ and $\mathbf{v} \times \mathbf{a} = \langle 0, -1, 1 \rangle$. Find $N$. 
