PROBLEM SET 3

The problems in this set take full advantage of the divergence theorem. In particular problem 1 leads you to compute a volume using surface integrals, while problem 2 leads you to find the flux through a non-closed surface by combining the divergence theorem with a much simpler surface integral.

**Problem 1.** Let $S$ be the surface obtained by rotating the curve

$$z = \sin(2t), \quad r = \cos(t), \quad -\pi/2 \leq t \leq \pi/2$$

about the $z$-axis. Find the volume of the solid $T$ whose boundary is $S$.

*Hint: Setting up the volume triple integral would be difficult. However by the divergence theorem we know the volume of $T$ is $\iiint_T dV = \iint_S \mathbf{F} \cdot \mathbf{n} dS$ where $\nabla \cdot \mathbf{F} = 1$. Here you have flexibility in choosing the vector field $\mathbf{F}$. To determine the choice which results in the simplest surface integral calculation first parametrize $S$ by $\mathbf{r}(u,v) = (x(u,v), y(u,v), z(u,v))$ by noting that the $r$ in the problem above is such that $x^2 + y^2 = r^2$. After computing the outer normal vector $\mathbf{N}(u,v)$ it should become clear which choice of $\mathbf{F}$ works best.*

**Problem 2.** Let $S$ be the outward oriented surface $x^2 + y^2 + z^2 = 4$ such that $z \leq 1$. Calculate the outward flux $\Phi$ given $\mathbf{F} = \langle e^z, xz, x^2 + y^2 \rangle$.

*Hint: Calculating the surface integral directly would require parametrizing $S$ using spherical coordinates, and then the expression for $\mathbf{F} \cdot \mathbf{n}$ would become very complicated. Instead construct a solid $T$ for which $S$ is part of the boundary. Pick $T$ so that the surface integral over the remainder of the boundary is easier to deal with. Apply the divergence theorem to $T$.**