**Stokes’ Theorem problem set - Solutions**

12/08/2015

**Problem 1**

By Stokes’ theorem, \( \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \nabla \times \mathbf{F} \cdot \mathbf{n} \, dS \), where \( S \) is the surface of the triangle produced by \( 2x + y + 2z = 2 \) in the first octant, with outward normal. The curl of \( \mathbf{F} \) is \((-2z, -2x, -2y)\). For \( S \) we have the parametrization \( \mathbf{r} = xi + yj + \frac{2-2x-y}{2}k \), so \( \mathbf{r}_x \times \mathbf{r}_y = i + \frac{1}{2}j + k \). Thus the double integral is \( \iint_T 2x + y - 2 \cdot x - 2ydydx = \iint_T x - y - 2dydx \), where \( T \) is the projection into the \( xy \) plane of the surface \( S \). This projection is a triangle with vertices \((0, 0), (0, 2), (1, 2)\), so the integral is \( \int_0^1 \int_0^{2-2x} x - y - 2dydx = -\frac{7}{3} \).

**Problem 2**

We can apply Stokes’ theorem. The curl of the vector field is \( \nabla \times \mathbf{F} = (-2z, -3x^2, -1) \). Following the hint, notice that the surface enclosed by the curve is parametrized by \( \mathbf{r} = xi + yj + 2xyk \). Hence \( \mathbf{r}_x \times \mathbf{r}_y = -2yi - 2xj + k \). Note that the curve \( C \) is oriented clockwise, which implies that the normal should point inward. Therefore, \( \mathbf{N} = 2yi + 2xj - k \) Thus \( \iint_S \nabla \times \mathbf{F} \cdot n \, dS = \iint_D -8xy^2 - 6x^3 + 1 \, dS \), where \( D \) is the disk \( x^2 + y^2 = 1 \). Switching to polar coordinates and integrating gives the answer \( \pi \).