1. (a) (5 pts) Determine whether the four points $P(2, 0, 0)$, $Q(1, 0, 3)$, $R(3, -1, -1)$, and $S(0, 1, 2)$ are coplanar, i.e. lie on the same plane (with complete justification).

ANSWER:

WORK:

(b) (5 pts) Find an equation of the form $Ax + By + Cz = D$ for the plane passing through the point $P = (1, 2, 2)$ and perpendicular to the line given in symmetric form as

$$\frac{x - 2}{3} = \frac{y - 3}{4} = z - 5.$$

ANSWER:

WORK:
2. (10 pts) Find and classify the critical points of the function \( f(x, y) = x^2 - xy^2 + 4x \).

ANSWER:

WORK:
3. Consider the function \( f(x, y) = \cos x + y^2 \sin x \) and the point \( P = (0, 3) \).

(a) (5pts) Find the gradient \( \nabla f(P) \).

ANSWER:

WORK:

(b) (5pts) Write the linear approximation formula for \( f(x, y) \) near \( f(P) \), and write the approximate value it gives for \( f(0.1, 2.9) \).

ANSWER:

WORK:

(c) (5pts) For an arbitrary unit vector \( u = (a, b) \), write a formula for the directional derivative \( D_u f(P) \).

ANSWER:

WORK:
4. (15 pts) Let \( f(x, y, z) \) be any function which only depends on the distance to the origin, so \( f(x, y, z) = f(\rho) \), where \( \rho^2 = x^2 + y^2 + z^2 \). Show that

\[
\left( \frac{df}{d\rho} \right)^2 = \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 + \left( \frac{\partial f}{\partial z} \right)^2
\]

ANSWER:

WORK:
5. (a) (15 pts) Using Langrange multipliers show that the maximum value attained by the function

\[ f(x, y, z) = x + y + z \]

at points of the sphere \( x^2 + y^2 + z^2 = a^2 \) is \( a\sqrt{3} \).

ANSWER:

WORK:

(b) (5 pts) Conclude from the result in part (a) that the arithmetic mean is no greater than the root-square mean, i.e. prove that

\[ \frac{x + y + z}{3} \leq \sqrt{\frac{x^2 + y^2 + z^2}{3}} \]

ANSWER:

WORK:
6. (10 pts) Evaluate

\[
\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{x^2} e^{y^2} \, dx \, dy.
\]

**ANSWER:**

**WORK:**
7. (20 pts) Explicitly evaluate the line integral

$$\int_C yz \, dx + x^2 \, dy + z \, dz,$$

where $C$ is the curve parametrized by $x = 2$, $y = \log t$, and $z = e^t$ with $1 \leq t \leq 2$.

**ANSWER:**

**WORK:**
8. Consider the vector field

\[ \mathbf{F} = (2xyz + 2xe^{xz}) \mathbf{i} + (x^2z + \cos y) \mathbf{j} + (x^2y + 2xe^{xz}) \mathbf{k}. \]

(a) (10 pts) Prove that \( \mathbf{F} \) is irrotational and find a potential function.

**ANSWER:**

**WORK:**

(b) (10 pts) Apply the Fundamental Theorem to compute the line integral

\[ \int_C \mathbf{F} \cdot T \, ds \]

where \( C \) is any curve starting at \( P(0, \pi, \sqrt{2008}) \) and ending at \( Q(e^{-2008}, \pi/2, 0) \).

**ANSWER:**

**WORK:**
9. (20 pts) Explicitly compute the surface integral
\[ \iiint_S 4z(x^2 + y^2) \, dS, \]
where \( S \) is the unit upper hemisphere with upward pointing normal vector.

**ANSWER:**

**WORK:**

10. (20 pts) Apply Green's Theorem to compute the flux \( \oint_C \mathbf{F} \cdot \mathbf{n} \, ds \) of the vector field \( \mathbf{F} = \langle xy, \frac{1}{2}y^2 \rangle \) across the closed curve \( C \) bounded by the parabola \( x = y^2 \) and line \( y = x \).

ANSWER:

WORK:
11. (20 pts) Apply the Divergence Theorem to compute the flux $\iint_S \mathbf{F} \cdot \mathbf{n} dS$ of the vector field \[ \mathbf{F} = zxi + \log x \sin zj + e^y k \]
across the closed surface which is the boundary of the solid in the first octant bounded by the paraboloid $z = 4 - x^2 - y^2$ and coordinate planes. *(Hint: Try cylindrical coordinates.)*

**ANSWER:** (Simple as possible please.)

**WORK:**
12. (20 pts) Apply Stokes' Theorem to compute the work $\oint_C \mathbf{F} \cdot \mathbf{T} \, ds$ done by the vector field

$$\mathbf{F} = (zx, y, e^z \cdot x)$$

in moving a particle along the counter-clockwise oriented closed curve $C$ which is the intersection of the paraboloid $z = x^2 + y^2$ with the plane $x + z = 2$.

**ANSWER:**

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**WORK:**

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Extra credit: (5 pts) Does the answer imply that $F$ is conservative? Why or why not?