You may use one page of written notes but no other notes, texts, or information sources. No calculators, computers, web resources. Be neat! Circle your answer. Show your work. Use sentences to explain your work where appropriate.

1. In 5 consecutive tosses of a fair coin, what is the probability of getting a run of (at least) three heads in a row?

2. Show that if $X$ is uniform in $[0, 1]$ and $Y = -\log X$, then $Y$ is an exponential, mean 1 random variable. If $X$ is then conditioned to lie in the interval $[1/2, 1]$, what can we say about $Y = -\log X$?

3. How many times, on average, should you expect to have to roll a fair die in order for every number to appear at least once? (Hint: assuming you’ve already seen $j$ different numbers, how long until the $j + 1$st number appears?)

4. How many tosses of a fair coin should you plan on making so that the probability of getting at least 55 heads is greater than .998? Use a normal approximation, and the fact that $0.02 \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-2} e^{-x^2/2} dx$.

5. Two urns are each filled with 1000 marbles; urn 1 has 50 blues and the remaining white, and urn 2 has 100 blues and the remaining white. You select an urn at random and draw 20 marbles. Using a Poisson approximation for both urns, what is the probability of drawing exactly 2 blues? Assuming you draw exactly 2 blues out of the 20, what is the probability that the urn selected was the first urn?

6. A distribution on $\{0, 1, 2\}$ has mean $2/3$ and variance $5/9$. What is the third moment?

7. $X$ and $Y$ are random variables with exponential distributions with parameters $a$ and $b$ respectively. Let $Z = \min\{X, Y\}$. Show that $Z$ is exponential with parameter $a + b$.

8. Joe is playing egg-tossing with Jane. Joe drops the egg with probability $a$, and Jane with probability $b$. (This Markov chain is illustrated in the graph below). How many tosses will the game last, on average, if Joe starts with the egg?