1. How many dimer configurations are there on a grid on an $m \times n$ cylinder, that is, a graph obtained from the $(m + 1) \times n$ grid by identifying vertices on the left and right by translation.

2. For $m, n$ both even take an $(m+1)\times n$ square grid and add diagonal edges on the faces of the last column, between $(m, y)$ and $(m+1, y+1)$. Show that there is a 1-to-1 mapping from dimer covers of this graph to dimer covers of the $m \times n$ grid with “free” boundary conditions on the right edge, that is, in which the vertices on the right edge do not all need to be used.

3. Take a $2 \times 2$ grid on the torus. Fix a dimer cover $m_0$. Given any other dimer cover, superposition with $m_0$ gives a homology class in $\mathbb{Z}^2$. What is the set of homology classes of dimer covers? Do the same for a $2 \times 4$ grid on a torus.

4. Let $G = (V, E)$ be a finite bipartite graph. A fractional matching is a function $f : E \rightarrow [0, 1]$ with the property that the sum at each vertex is 1. The set of fractional matchings is thus a polytope in $[0, 1]^E$. Show that its vertices are exactly the dimer covers of $G$.

5. Find the characteristic polynomial for the square grid dimer with edge weights $a, b, c, d$ on edges around the white vertices in directions N,W,S,E respectively.