6. Verify that at any point \((x, y, z) \in S^2\) the vectors \((-y, x, 0)\) and \((xz, yz, z^2 - 1)\) are tangent to \(S^2\). Write these same tangent vectors down in terms of the (stereographic) coordinates \(u = \frac{x}{1 + z}\), \(v = \frac{y}{1 + z}\) defined in \(S^2 - \{(0, 0, -1)\}\). The answers should have the form \(a \frac{\partial}{\partial u} + b \frac{\partial}{\partial v}\) with \(a\) and \(b\) being functions of \(u\) and \(v\).

7. Define \(F : \mathbb{R}^2 \to \mathbb{R}^3\) by \(F(x, y) = (x, y^2, y^3 - xy)\). Find all points in \(\mathbb{R}^2\) at which \(F\) fails to be an immersion. Find all points \(p \in \mathbb{R}^3\) such that there is more than one point \(q \in \mathbb{R}^2\) for which \(F(q) = p\). For each such \(p\), determine all of the “tangent planes of \(F(\mathbb{R}^2)\) at \(p\)” (the images of the linear maps \(D_qF : \mathbb{R}^2 \to \mathbb{R}^3\)).

8. Show that there is a smooth embedding of \(\mathbb{R}P^n\) in the space of real \((n+1) \times (n+1)\) real matrices, defined by \(e(x_0 : \ldots : x_n) = \frac{1}{|x|^2} x^t x\). Here \(x = (x_0, \ldots, x_n)\) is a nonzero vector in \(\mathbb{R}^{n+1}\), considered also as a \(1 \times (n+1)\) matrix, and \(x^t\) is the transpose.