March 2015
MA 2110, Introduction to Manifolds
Fifth Assignment
Due 3/19/15

March 18, 2015

A diagonal point in $M \times M$ is a point $(p, q)$ such that $p = q$. Denote the set of diagonal points by $\Delta_M$. For $F : M \to N$ a double point is a point $(p, q)$ such that $p \neq q$ and $F(p) = F(q)$. Thus if we write $F^{(2)} : M \times M - \Delta_M \to N \times N$ for the restriction of the map $F \times F : M \times M \to N \times N$ then the set of double points is the preimage of $\Delta_N$ under $F^{(2)}$. Say that the double points of $F$ occur transversely if $F^{(2)}$ is transverse to $\Delta_N$. Note that in this case the set of double points is a smooth manifold of dimension $2m - n$ (in particular empty if $2m < n$).

12. Show that the map $F \times F$ can never be transverse to $\Delta_N$ at a diagonal point of $M \times M$ if $m < n$.

A non-immersion point of $F$ is a point $p \in M$ such that the linear map $D_pF : T_pM \to T_{F(p)}N$ is not injective.

13. Prove that if the diagonal point $(p, p)$ is a limit of double points for $F$ then $p$ is a non-immersion point for $F$.

Let $0_M \subset TM$ be the image of the zero-section $M \to TM$. Denote by $D^{(0)}F$ the restriction of $DF : TM \to TN$ to the complement $TM - 0_M$, so that $F$ is an immersion if and only if the inverse image of $0_N$ by $D^{(0)}F$ is empty. Say that the non-immersion points of $F$ occur transversely if $D^{(0)}F$ is transverse to $0_N$.

14. Show that $F$ must be an immersion if the non-immersion points occur transversely and $2m \leq n$.

15. Show that for the map $\mathbb{R}^2 \to \mathbb{R}^3$ of Problem 7 both double points and non-immersion points occur transversely.